

**MATH 360**  
**HOMEWORK 2**

(due to Feb 6)

1. Show that 1 is not a limit of the sequence  $x_n = (-1)^n$ . (5 pts)
  
2. The sequence  $\{x_n\}$  is defined by the recurrence  $x_{n+1} = \frac{x_n}{2} + \frac{1}{x_n}$  and the initial condition  $x_1 = 2$ .
  - a) Show that it converges. (4 pts)
  - b) Find its limit. (2 pts)
  
3. Find all cluster points for the sequences:
  - a)  $x_n = n$ ; (1pt)                      b)  $x_n = \frac{1}{n}$ ; (1pt)
  - c)  $x_n = \sin \frac{\pi n}{6}$ ; (2pts)                      d)  $x_n = n$ -th rational number. (5pts)

In the last problem a labelling of rational numbers by positive integers is used. (Such labellings exist because  $\mathbb{Q}$  is denumerable and we fix one of them; the answer does not depend on this choice.)