11.Determine whether the **improper** integral

$$\int_0^{\infty} \frac{1}{xe^x} dx$$

is convergent or not. (10 points)

① Since 0 is ~~a~~ the discontinuous point of $\frac{1}{xe^x}$, $\int_0^1 \frac{1}{xe^x} dx$ is ~~both~~ an infinite integral as well as a discontinuous integral

~~②~~ ③ $\int_0^{\infty} \frac{1}{xe^x} dx = \int_0^1 \frac{1}{xe^x} dx + \int_1^{+\infty} \frac{1}{xe^x} dx$

③ Look at $\int_0^1 \frac{1}{xe^x} dx$

For $0 \le x \le 1$

$0 < 1 \le e^x \le e$

so $\frac{1}{e} \le \frac{1}{e^x}$ ~~[illegible]~~

so $\frac{1}{e}\frac{1}{x} \le \frac{1}{xe^x}$ $0 < x \le 1$

We know $\int_0^1 \frac{1}{e}\frac{1}{x} dx$ is divergent

by comparision test $\int_0^1 \frac{1}{xe^x} dx$ is divergent.

④ ~~⑤~~ So $\int_0^{\infty} \frac{1}{xe^x} dx$ is $\boxed{\text{divergent}}$

we dont need to check $\int_1^{\infty} \frac{1}{xe^x} dx$