7. Determine the convergence or divergence of the **series**

$$\sum_{n=1}^{\infty} \frac{2n+3}{n!}$$

where $n! = n \cdot (n-1) \cdot \cdots \cdot 2 \cdot 1$. (10 points)

① Notice that $\frac{2n+3}{n!} = \frac{2n}{n!} + \frac{3}{n!}$

For $n \geqslant 2$

$$n! = n \cdot (n-1) \cdot \cdots \cdot 2 \cdot 1$$
$$= n \cdot (n-1)!$$

$$\cancel{= \frac{2n}{n \cdot (n-1)!} + \frac{3}{n!}}$$
$$\cancel{= \frac{2}{(n-1)!} + \frac{3}{n!}}$$

so for $n \geqslant 2$

$$\frac{2n}{n!} = \frac{2n}{n(n-1)!} = \frac{2}{(n-1)!}$$

so for $n \geqslant 2$ $\quad \frac{2n+3}{n!} = \frac{2}{(n-1)!} + \frac{3}{n!}$

② We can show that $\sum_{n=2}^{\infty} \frac{2}{(n-1)!}$ and $\sum_{n=2}^{\infty} \frac{3}{n!}$

are both convergent

For $\sum_{n=2}^{\infty} \frac{3}{n!}$, notice for $n \geqslant 2$

$$n! \geqslant n(n-1)$$

$$\text{so } 0 \leq \frac{3}{n!} \leq \frac{3}{n(n-1)}$$

and $\sum_{n=2}^{\infty} \frac{3}{n(n-1)}$ is convergent,

so by comparision theorem, $\sum_{n=1}^{\infty} \frac{3}{n!}$ is

convergent.