

9. Do the indefinite integral

$$\int \frac{x+1}{\sqrt{9-x^2}} dx$$

(10 points)

$$\textcircled{1} \quad \int \frac{x+1}{\sqrt{9-x^2}} dx = \int \frac{x}{\sqrt{9-x^2}} dx + \int \frac{1}{\sqrt{9-x^2}} dx$$

$$\textcircled{2} \quad \int \frac{x}{\sqrt{9-x^2}} dx$$

$$u = 9 - x^2$$

$$du = -2x dx, \quad x dx = -\frac{1}{2} du$$

$$\begin{aligned} \text{so } \int \frac{x dx}{\sqrt{9-x^2}} &= \int -\frac{1}{2} \frac{du}{\sqrt{u}} \\ &= -\frac{1}{2} \cdot 2 \sqrt{u} + C \\ &= -\sqrt{u} + C \\ &= -\sqrt{9-x^2} + C \end{aligned}$$

$$\textcircled{3} \quad \int \frac{1}{\sqrt{9-x^2}} dx = \int \frac{1}{\sqrt{3^2-x^2}} dx$$

$$x = 3 \sin \theta \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

$$dx = 3 \cos \theta d\theta \quad \sqrt{3^2-x^2} = \sqrt{3^2-3^2 \sin^2 \theta} = 3|\cos \theta| = 3 \cos \theta$$

$$\begin{aligned} \text{so } \int \frac{1}{\sqrt{3^2-x^2}} dx &= \int \frac{3 \cos \theta d\theta}{3 \cos \theta} = \int 1 d\theta = \theta + C \end{aligned}$$