

12. Do the definite integral

$$\int_0^2 \sqrt{x^2 + 4x} dx$$

Hint: Maybe you should first complete the square in the square-root. (10 points)

$$\textcircled{1} \quad \sqrt{x^2 + 4x} = \sqrt{x^2 + 4x + 4 - 4} = \sqrt{(x+2)^2 - 4} \\ = \sqrt{(x+2)^2 - 2^2}$$

$$\textcircled{2} \quad \text{Let } x+2 = 2 \sec \theta$$

x from 0 to 2

$x+2$ ~~for~~ from 2 to 4

$$\sec \theta = \frac{x+2}{2} \quad \text{from 1 to 2}$$

$$\theta \text{ from } 0 \text{ to } \frac{\pi}{3}$$



$$\textcircled{3} \quad \sqrt{(x+2)^2 - 2^2} = \sqrt{(2 \sec \theta)^2 - 2^2} = \sqrt{2^2 \tan^2 \theta} \\ = 2 \tan \theta$$

$$dx = 2 \sec \theta \tan \theta d\theta$$

$$\textcircled{3} \quad \int_0^2 \sqrt{x^2 + 4x} dx = \int_0^{\frac{\pi}{3}} 2 \sec \theta \tan \theta \cdot 2 \tan \theta d\theta \\ = \int_0^{\frac{\pi}{3}} 4 \sec \theta \tan^2 \theta d\theta \\ = \int_0^{\frac{\pi}{3}} 4 \sec \theta (\sec^2 \theta - 1) d\theta$$