For $\sum_{n=2}^{\infty} \frac{2}{(n-1)!}$, re-index

let $m = n-1$

$$\sum_{n=2}^{\infty} \frac{2}{(n-1)!} = \sum_{m=1}^{\infty} \frac{2}{m!} = \sum_{n=1}^{\infty} \frac{2}{n!}$$

We know $\sum_{n=1}^{\infty} \frac{2}{n!}$ is convergent by the same reason as $\sum_{n=2}^{\infty} \frac{3}{n!}$, so $\sum_{n=2}^{\infty} \frac{2}{(n-1)!}$ is convergent

③ Finally

$$\sum_{n=2}^{\infty} \frac{2n+3}{n!} = \sum_{n=2}^{\infty} \frac{2}{(n-1)!} + \sum_{n=2}^{\infty} \frac{3}{n!}$$

is the sum of two convergent series

so $\sum_{n=2}^{\infty} \frac{2n+3}{n!}$ is convergent

so $\sum_{n=1}^{\infty} \frac{2n+3}{n!}$ is $\boxed{\text{convergent}}$.