1. Determine the convergence or divergence of the **series**

$$\sum_{n=1}^{\infty}[sin\left(\frac{1}{n}\right)]^3$$

(5 points)

limit comparision test

compare with $\sum_{n=1}^{\infty} \frac{1}{n^3}$

$$\lim_{n\to\infty} \frac{[\sin(\frac{1}{n})]^3}{\frac{1}{n^3}} = \lim_{n\to\infty} \left[\frac{\sin(\frac{1}{n})}{\frac{1}{n}}\right]^3$$

$$= \left[\lim_{n\to\infty} \frac{\sin(\frac{1}{n})}{\frac{1}{n}}\right]^3$$

$$= \left[\lim_{x\to 0} \frac{\sin x}{x}\right]^3$$

L'Hospital's rule

$$= \left[\lim_{x\to 0} \frac{(\sin x)'}{(x)'}\right]^3$$

$$= \left[\lim_{x\to 0} \frac{\cos x}{1}\right]^3$$

$$= 1^3 = 1 \neq 0$$

and $\sum_{n=1}^{\infty} \frac{1}{n^3}$ is convergent.

By limit comparision test, $\sum_{n=1}^{\infty} [\sin(\frac{1}{n})]^3$ is $\boxed{\text{convergent.}}$