3. Determine the convergence or divergence of the **series**

$$\sum_{n=1}^{\infty} \frac{n^2+5}{\sqrt{n^6+n}}$$

(5 points)

limit comparision test

Highest degree of n^2+5 is 2

Highest degree of $\sqrt{n^6+n}$ is $\frac{6}{2}=3$

so compare with $\sum_{n=1}^{\infty} \frac{n^2}{n^3} = \sum_{n=1}^{\infty} \frac{1}{n}$

$$\lim_{n\to\infty} \frac{\frac{n^2+5}{\sqrt{n^6+n}}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^3+5n}{\sqrt{n^6+n}}$$

$$= \lim_{n\to\infty} \frac{1+\frac{5}{n^2}}{\sqrt{1+\frac{n}{n^6}}}$$

$$= \frac{1+0}{\sqrt{1+0}} = 1 \neq 0$$

and $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent

By limit comparision test

$\sum_{n=1}^{\infty} \frac{n^2+5}{\sqrt{n^6+n}}$ is $\boxed{\text{divergent.}}$.