

PUTNAM PRACTICE PROBLEMS 6

Exercise 1. The Fibonacci sequence (F_n) is given by:

$$\begin{cases} F_1 := 1, F_2 := 1 \\ F_{n+2} := F_n + F_{n+1} \text{ for } n \geq 1. \end{cases}$$

Find a closed formula for F_n in terms of n .

Exercise 2. Given n a non-negative integer, find the largest power of 2 which divides $\lfloor (1+\sqrt{3})^{2n+1} \rfloor$. Here $\lfloor x \rfloor$ denotes the largest integer which is less than equal to x .

Exercise 3. Prove that there exists a unique function f mapping the set of positive real numbers into \mathbb{R} which satisfies:

- i) $f(f(x)) = 6x - f(x)$ for all $x > 0$.
- ii) $f(x) > 0$ for all $x > 0$.