# PUTNAM PRACTICE PROBLEMS 5

**Exercise 1.** *Given a triangle $\Delta ABC$, and a point $P$ inside the triangle, we denote by $d_a, d_b, d_c$ the distances from $P$ to the lines which contain the sides of the triangle. Find the point $P$ for which the product $d_a \cdot d_b \cdot d_c$ is maximized.*

**Exercise 2.** *Suppose that $x, y, z > 0$. Show that:*

$$\frac{x^3}{x^2+xy+y^2} + \frac{y^3}{y^2+yz+z^2} + \frac{z^3}{z^2+zx+x^2} \geq \frac{x+y+z}{3}$$

*Furthermore, show that equality holds if and only if $x = y = z$.*

**Exercise 3.** *Let $I_n := 1 + \frac{1}{2} + \cdot + \frac{1}{n}$. Without using the logarithmic asymptotics for $I_n$, show that:*

$$I_n \geq n(\sqrt[n]{n+1} - 1).$$