PUTNAM PRACTICE PROBLEMS 1

Exercise 1. *Suppose that P is a polynomial with integer coefficients and suppose that there exists a positive integer n such that none of the values $P(1), P(2), \ldots, P(n)$ are divisible by n. Show that P doesn't have any integer roots.*

Exercise 2. *Let n be a positive even integer. We write the numbers $1, 2, \ldots, n^2$ in a square grid such that the k-th row, from left to right reads:*

$$(k-1)n + 1, (k-1)n + 2, \ldots, (k-1)n + n.$$

We then color the squares on the grid so that half of the squares in each row and an each column are colored red and half are colored blue. Show that the sum of the numbers which are colored red is equal to the sum of the numbers which are colored blue.

Exercise 3. *Suppose that n is a positive integer. Prove that:*

$$\binom{n}{0}^2 + \binom{n}{1}^2 + \ldots + \binom{n}{n}^2 = \binom{2n}{n}.$$

Exercise 4. *Suppose that f is a non-negative continuous function on $\mathbb{R}$. Suppose that, for every $\epsilon \in [0, 1)$, one has $\lim_{n \in \mathbb{N}; n \to \infty} f(\epsilon + n) = 0$. Show by example, that $\lim_{x \to \infty} f(x)$ doesn't have to equal zero.*