MATH 425, HOMEWORK 1

This homework is due in class on Thursday, January 24. Each problem is worth 10 points.

Exercise 1. *We recall from class that an operator $\mathcal{L}$ acting on functions is said to be* ***linear*** *if for all functions u, v and for all scalars a, b, one has $\mathcal{L}(au + bv) = a \cdot \mathcal{L}u + b \cdot \mathcal{L}v$.*

Which of the following operators are linear?

a) $\mathcal{L}u = u_{xx} + u_{xy}$.
b) $\mathcal{L}u = u_t + uu_x$.
c) $\mathcal{L}u = \sin(x^2y)u_x + e^{xy^2}u_y$.
d) $\mathcal{L}u = u_x + u_y + 1$.
e) $\mathcal{L}u = u_{xx} + \sin(u)$.
Give a brief justification for each answer.

In the following exercises, u is assumed to be a function of two variables.

Exercise 2. *(Strauss, Exercise 1.2.1.)*
Solve the first order PDE: $2u_t + 3u_x = 0$, *with the auxiliary condition* $u = \sin x$ *when* $t = 0$.

Exercise 3. *(Strauss, Exercise 1.2.3.)*
Solve the equation: $(1 + x^2)u_x + u_y = 0$. *Describe its characteristic curves.*

Exercise 4. *(Strauss, Exercise 1.2.6.)*
a) Solve the equation: $yu_x + xu_y = 0$, *with the condition* $u(0, y) = e^{-y^2}$.
b) In which region of the xy*-plane is the solution uniquely determined?*

Exercise 5. *(Strauss, Exercise 1.2.11.)*
Use the coordinate method in order to solve the equation:

$$u_x + 2u_y + (2x - y)u = 2x^2 + 3xy - 2y^2.$$