Math 114, Section 003 Fall 2011
Practice Exam 1 with Solutions

Contents

1 Problems

Question 1: Let L be the line tangent to the curve

$$\overrightarrow{r}(t) = \left\langle t^2 + 3t + 2, e^t \cos t, \ln(t+1) \right\rangle$$

at $t = 0$. Find the coordinates of the point of intersection of L and the plane $x + y + z = 8$.

(A) $(2, 1, 0)$ (B) $(6, e^2 \cos 2, \ln(2))$ (C) $(2, 0, 1)$
(D) $(5, 2, 1)$ (E) $(8, 3, 2)$ (F) $(0, 4, 4)$

SOLUTION KEY: 2.1 SOLUTION: 3.1

Question 2: Which vector is perpendicular to the plane containing the three points $P(2, 1, 5)$, $Q(-1, 3, 4)$, and $R(3, 0, 6)$?

(A) $2\widehat{\boldsymbol{\imath}} - \widehat{\boldsymbol{\jmath}} + \widehat{\boldsymbol{k}}$ (B) $\widehat{\boldsymbol{\imath}} + 2\widehat{\boldsymbol{\jmath}} + 2\widehat{\boldsymbol{k}}$ (C) $2\widehat{\boldsymbol{\imath}} + 2\widehat{\boldsymbol{\jmath}} - \widehat{\boldsymbol{k}}$
(D) $2\widehat{\boldsymbol{\imath}} + 3\widehat{\boldsymbol{\jmath}} + \widehat{\boldsymbol{k}}$ (E) $\widehat{\boldsymbol{\imath}} + 2\widehat{\boldsymbol{\jmath}} + \widehat{\boldsymbol{k}}$ (F) $2\widehat{\boldsymbol{\imath}} + 2\widehat{\boldsymbol{\jmath}} + 3\widehat{\boldsymbol{k}}$

SOLUTION KEY: 2.2 SOLUTION: 3.2

Question 3: A force $\overrightarrow{F} = 2\widehat{\boldsymbol{\imath}} + \widehat{\boldsymbol{\jmath}} - 3\widehat{\boldsymbol{k}}$ is applied to a spacecraft with velocity vector $\overrightarrow{v} = 3\widehat{\boldsymbol{\imath}} - \widehat{\boldsymbol{\jmath}}$. If you express $\overrightarrow{F} = \overrightarrow{a} + \overrightarrow{b}$ as a sum of a vector $\overrightarrow{a}$ parallel to $\overrightarrow{v}$ and a vector $\overrightarrow{b}$ orthogonal to $\overrightarrow{v}$, then $\overrightarrow{b}$ is:

(A) $3\widehat{\boldsymbol{\imath}} + 6\widehat{\boldsymbol{\jmath}} - \widehat{\boldsymbol{k}}$ (B) $\frac{1}{2}\widehat{\boldsymbol{\imath}} + \frac{3}{2}\widehat{\boldsymbol{\jmath}} - 3\widehat{\boldsymbol{k}}$ (C) $\frac{3}{2}\widehat{\boldsymbol{\imath}} - \frac{3}{2}\widehat{\boldsymbol{\jmath}}$

(D) $8\widehat{\boldsymbol{\imath}} + 4\widehat{\boldsymbol{\jmath}} - \widehat{\boldsymbol{k}}$ (E) $12\widehat{\boldsymbol{\imath}} - \widehat{\boldsymbol{\jmath}} + \widehat{\boldsymbol{k}}$ (F) $\widehat{\boldsymbol{\imath}} + 3\widehat{\boldsymbol{\jmath}} - \widehat{\boldsymbol{k}}$

SOLUTION KEY: 2.3 SOLUTION: 3.3

Question 4: Which of the following points lies on the same plane with $(1, 2, 0)$, $(2, 2, 1)$, $(0, 1, 1)$?

(A) $(1, 1, 1)$ (B) $(4, -1, 1)$ (C) $(4, 1, 1)$

(D) $(1, 1, -1)$ (E) $(2, -1, 3)$ (F) $(2, 1, 3)$

SOLUTION KEY: 2.4 SOLUTION: 3.4

Question 5: The curvature of the curve

$$\overrightarrow{r}(t) = 2t\,\widehat{\boldsymbol{\imath}} + t^2\,\widehat{\boldsymbol{\jmath}} - \frac{1}{3}\,\widehat{\boldsymbol{k}}$$

at the point $t = 0$ is

(A) 0 (B) 2 (C) $-\frac{1}{4}$

(D) $\frac{1}{2}$ (E) -1 (F) none of the above

Solution Key: 2.5 Solution: 3.5

Question 6: Which of the following surfaces intersect the plane $x = 2$ at a parabola?

(A) $-\frac{z^2}{2} = \frac{x^2}{9} + \frac{y^2}{4}$ (B) $\frac{z^2}{4} = \frac{x^2}{9} + \frac{y^2}{4} - 1$ (C) $\frac{z^2}{4} = \frac{x^2}{9} + \frac{y^2}{4}$

(D) $\frac{z}{2} = \frac{x^2}{9} - \frac{y^2}{4}$ (E) $\frac{z^2}{4} = \frac{x^2}{9} + \frac{y^2}{4} + 1$ (F) $-\frac{z^2}{25} = \frac{x^2}{9} + \frac{y^2}{4}$

Solution Key: 2.6 Solution: 3.6

Question 7: The set of points P such that $\overrightarrow{QP} \cdot \overrightarrow{B} = 2$ is

(A) a line though Q parallel to $\overrightarrow{B}$

(B) a plane through Q parallel to $\overrightarrow{B}$

(C) a plane through Q perpendicular to $\overrightarrow{B}$

(D) a line parallel to $\overrightarrow{B}$ but not passing through Q

(E) a plane perpendicular to $\overrightarrow{B}$ but not passing through Q

(F) a line through Q and perpendicular to $\overrightarrow{B}$

Solution Key: 2.7 Solution: 3.7

Question 8: The trout in a pond is harvested at a constant rate of H trout per day. It is known taht the growth of the trout population is governed by the logistic equation with harvesting:

$$\frac{dP}{dt} = 12P - P^2 - H.$$

(a) For what harvesting rates will this growth model have two equilibrium populations? Will these equilibria be stable or unstable?

(b) Determine the special value of H for which the population growth has a single equilibrium. What will happen if we start harvesting at a rate higher than this special value?

SOLUTION KEY: 2.8 SOLUTION: 3.8

Question 9: True or false. Explain your reasoning.

(a) The line $\vec{r}(t) = (1+2t)\hat{\boldsymbol{\imath}} + (1+3t)\hat{\boldsymbol{\jmath}} + (1+4t)\hat{\boldsymbol{k}}$ is perpendicular to the plane $2x + 3y - 4z = 9$.

(b) The equation $x^2 = z^2$ in three dimensons, describes an ellipsoid.

(c) $|\vec{a} \times \vec{b}| = 0$ implies that either $\vec{a} = 0$ or $\vec{b} = 0$.

SOLUTION KEY: 2.9 SOLUTION: 3.9

Question 10: A bug is crawling along a helix and his position at ttime t is given by $\overrightarrow{r}(t) = \langle \sin(2t), \cos(2t), t \rangle$. Which of the following statements are true and which are false? Explain and justify your reasoning.

(a) The unit normal vector always points toward the z-axis.

(b) The bug travels upward at a constant rate, i.e. the unit tangent vector has a constant z-component at any moment of time.

(c) The unit binormal vector always points straight up or straight down.

Solution Key: 2.10 Solution: 3.10

2 Solution key

(1) (D)

(2) (E)

(3) (B)

(4) (F)

(5) (D)

(6) (D)

(7) (E)

(8) **(a)** $H < 36$, one unstable and one stable equilibrium; **(b)** to have one equilibrium we must harvest at a rate $H = 36$. If $H > 36$, then the trout population will go extinct.

(9) (a), (b), and (c) are false

(10) (a) is true, (b) is true, and (c) is false.

3 Solutions

Solution of problem 1.1: The point P on the curve corresponding to the value of the parameter $t = 0$ has position vector $\overrightarrow{r}(0) = \langle 2, 1, 0\rangle$. In other words P has coordinates $P(2, 1, 0)$. The tangent vector to the curve at P is the vector

$$\begin{aligned}\frac{d\overrightarrow{r}}{dt}(0) &= \left(\frac{d}{dt}\left\langle t^2 + 3t + 2, e^t \cos t, \ln(t + 1)\right\rangle\right)_{|t=0} \\ &= \left(\left\langle 2t + 3, e^t \cos t - e^t \sin t, \frac{1}{t + 1}\right\rangle\right)_{|t=0} \\ &= \langle 3, 1, 1\rangle.\end{aligned}$$

The tangent line L at $t = 0$ is the line passing through P and having the tangent vector $d\overrightarrow{r}/dt(0)$ as a direction vector. Thus L is given by the parametric equations

$$\begin{aligned}x &= 2 + 3s,\\ y &= 1 + s,\\ z &= s.\end{aligned}$$

To intersect the line L with the plane $x + y + z = 8$ we substitute the parametric expressions for x, y, and z in the equation of the plane and solve for s. We get $(2+3s)+(1+s)+s = 8$, i.e. $s = 1$. Thus the point of intersection is the point on the line which corresponds to the value of the parameter $s = 1$, i.e. the point $(5, 2, 1)$. The correct answer is (D).

Solution of problem 1.2: A vector is perpendicular to a plane if and only if it is parallel to a normal vector for the plane.

To find a vector $\overrightarrow{n}$ which is normal to the plane containing $P(2, 1, 5)$, $Q(-1, 3, 4)$, and $R(3, 0, 6)$ we need to find two non-parallel vectors in the plane and compute their cross products. The vectors

$$\overrightarrow{PQ} = \langle -1 - 2, 3 - 1, 4 - 5\rangle = \langle -3, 2, -1\rangle$$

$$\overrightarrow{PR} = \langle 3 - 2, 0 - 1, 6 - 5\rangle = \langle 1, -1, 1\rangle$$

are not parallel and belong to the plane, so we have

$$\overrightarrow{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \widehat{\boldsymbol{\imath}} & \widehat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ -3 & 2 & -1 \\ 1 & -1 & 1 \end{vmatrix} = (2-1)\widehat{\boldsymbol{\imath}} + (3-1)\widehat{\boldsymbol{\jmath}} + (3-2)\widehat{\boldsymbol{k}}$$

$$= \boxed{\widehat{\boldsymbol{\imath}} + 2\widehat{\boldsymbol{\jmath}} + \widehat{\boldsymbol{k}}}$$

The correct answer is (E).

Solution of problem 1.3: If $\overrightarrow{F} = \overrightarrow{a} + \overrightarrow{b}$ with $\overrightarrow{a} || \overrightarrow{v}$ and $\overrightarrow{b} \perp \overrightarrow{v}$, then $\overrightarrow{a}$ is the orthogonal projection of $\overrightarrow{F}$ onto $\overrightarrow{v}$. From the formula for an orthogonal projection we compute

$$\begin{aligned} \overrightarrow{a} &= \mathsf{proj}_{\overrightarrow{v}} \ \overrightarrow{F} \\ &= \frac{\overrightarrow{F} \cdot \overrightarrow{v}}{|\overrightarrow{v}|} \overrightarrow{v} \\ &= \frac{\langle 2, 1, -3 \rangle \cdot \langle 3, -1, 0 \rangle}{3^2 + (-1)^2} \langle 3, -1, 0 \rangle \\ &= \frac{5}{10} \langle 3, -1, 0 \rangle \\ &= \left\langle \frac{3}{2}, -\frac{1}{2}, 0 \right\rangle . \end{aligned}$$

Therefore

$$\begin{aligned} \overrightarrow{b} &= \overrightarrow{F} - \overrightarrow{a} \\ &= \langle 2, 1, -3 \rangle - \left\langle \frac{3}{2}, -\frac{1}{2}, 0 \right\rangle \\ &= \left\langle \frac{1}{2}, \frac{3}{2}, -3 \right\rangle . \end{aligned}$$

The correct answer is (B). □

Solution of problem 1.4: If we label the points on the plane as $P_0 = (1, 2, 0)$, $Q_0 = (2, 2, 1)$, and $R_0 = (0, 1, 1)$, then we can easily write two vectors parallel to the plane, e.g.

$$\vec{u} = \overrightarrow{P_0Q_0} = (2-1)\widehat{\boldsymbol{\imath}} + (2-2)\widehat{\boldsymbol{\jmath}} + (1-0)\widehat{\boldsymbol{k}} = \widehat{\boldsymbol{\imath}} + \widehat{\boldsymbol{k}}$$
$$\vec{v} = \overrightarrow{P_0R_0} = (0-1)\widehat{\boldsymbol{\imath}} + (1-2)\widehat{\boldsymbol{\jmath}} + (1-0)\widehat{\boldsymbol{k}} = -\widehat{\boldsymbol{\imath}} - \widehat{\boldsymbol{\jmath}} + \widehat{\boldsymbol{k}}.$$

The normal vector to the plane is given by the cross-product $\vec{\boldsymbol{n}} = \vec{u} \times \vec{v}$. We compute

$$\vec{\boldsymbol{n}} = \begin{vmatrix} \widehat{\boldsymbol{\imath}} & \widehat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ 1 & 0 & 1 \\ -1 & -1 & 1 \end{vmatrix} = \widehat{\boldsymbol{\imath}} - 2\widehat{\boldsymbol{\jmath}} - \widehat{\boldsymbol{k}}.$$

Thus a point $P = (x, y, z)$ on the plane must satisfy the equation $\vec{\boldsymbol{n}} \cdot \overrightarrow{P_0P} = 0$, which is

$$(x-1) - 2(y-2) - (z-0) = 0$$

or simply

$$x - 2y - z = -3.$$

Substituting the various choices in this equation we see that the only solution is the point $(2, 1, 3)$ corresponding to answer (F).

Solution of problem 1.5: The curvature of $\overrightarrow{r}(t)$ is given by

$$\kappa(t) = \frac{\left|\frac{d\overrightarrow{T}}{dt}\right|}{|\overrightarrow{r}'|},$$

where $\overrightarrow{T}$ is the unit tangent vector. We compute

$$\overrightarrow{r}'(t) = \langle 2, 2t, 0 \rangle,$$

and so $|\overrightarrow{r}'| = \sqrt{4 + 4t^2}$. In particular we have

$$\overrightarrow{T} = \left\langle 2(4+4t^2)^{-\frac{1}{2}}, 2t(4+4t^2)^{-\frac{1}{2}}, 0 \right\rangle.$$

Substituting in the formula for the curvature we get

$$\kappa(t) = \frac{\left|\frac{d\overrightarrow{T}}{dt}\right|}{|\overrightarrow{r}'|}$$

$$= \frac{\left|\frac{d}{dt}\left\langle 2(4+4t^2)^{-\frac{1}{2}}, 2t(4+4t^2)^{-\frac{1}{2}}, 0\right\rangle\right|}{(4+4t^2)^{\frac{1}{2}}}$$

$$= \frac{\left|\left\langle -8t(4+4^2)^{-\frac{3}{2}}, 2(4+4t^2)^{-\frac{1}{2}} - 8t^2(4+4^2)^{-\frac{3}{2}}, 0\right\rangle\right|}{(4+4t^2)^{\frac{1}{2}}}$$

Evaluating at $t = 0$ we get

$$\kappa(0) = \frac{|\langle 0, 1, 0\rangle|}{2} = \frac{1}{2}.$$

The correct answer is (D). □

Solution of problem 1.6: The curve of intersection of each surface with the plane $x = 2$ will be given by the equation in the variables y and z that is obtained from the equation of the surface after the substitution $x = 2$.

Substituting $x = 2$ in the equation of each surface we get the following equations in y and z:

(A) Setting $x = 2$ in $-\frac{z^2}{2} = \frac{x^2}{9} + \frac{y^2}{4}$ gives

$$-\frac{z^2}{2} = \frac{4}{9} + \frac{y^2}{4}$$

or equivalently

$$\frac{y^2}{4} + \frac{z^2}{2} = -\frac{4}{9}.$$

This equation has no solution so it describes the empty set. In other words the surface (A) and the plane $x = 2$ do not intersect.

(B) Setting $x = 2$ in $\dfrac{z^2}{4} = \dfrac{x^2}{9} + \dfrac{y^2}{4} - 1$ gives

$$\frac{z^2}{4} = \frac{4}{9} + \frac{y^2}{4} - 1$$

or equivalently

$$\frac{y^2}{4} - \frac{z^2}{4} = \frac{5}{9}$$

which is a scaling of a standard equation of a hyperbola.

(C) Setting $x = 2$ in $\dfrac{z^2}{4} = \dfrac{x^2}{9} + \dfrac{y^2}{4}$ gives

$$\frac{z^2}{4} = \frac{4}{9} + \frac{y^2}{4}$$

or

$$\frac{y^2}{4} - \frac{z^2}{4} = \frac{4}{9}$$

which is again a scaling of a standard equation of a hyperbola.

(D) Setting $x = 2$ in $\dfrac{z}{2} = \dfrac{x^2}{9} - \dfrac{y^2}{4}$ gives

$$\frac{z}{2} = \frac{4}{9} - \frac{y^2}{4}$$

or

$$z = \frac{8}{9} - \frac{y^2}{2}$$

which is the equation of a parabola.

(E) Setting $x = 2$ in $\dfrac{z^2}{4} = \dfrac{x^2}{9} + \dfrac{y^2}{4} + 1$ gives

$$\frac{z^2}{4} = \frac{4}{9} + \frac{y^2}{4} + 1$$

or

$$\frac{z^2}{4} - \frac{y^2}{4} = \frac{13}{9}$$

which is again a scaling of a standard equation of a hyperbola.

The correct answer is (D). □

Solution of problem 1.7: Given a point Q and a vector $\overrightarrow{B}$ the vector equation

$$\overrightarrow{QP} \cdot \overrightarrow{B} = 0$$

is the standard equation of the plane α that passes through Q and is perpendicular to $\overrightarrow{Q}$. The equation $\overrightarrow{QP} \cdot \overrightarrow{B} = 2$ will describe a plane β which is parallel to α and thus perpendicular to $\overrightarrow{B}$. Since the right hand side in this equation is $2 \neq 0$ we conclude that β does not pass through Q.

More explicitly, if $Q = (x_0, y_0, z_0)$ is a fixed point and $\overrightarrow{B} = a\widehat{\boldsymbol{\imath}} + b\widehat{\boldsymbol{\jmath}} + c\widehat{\boldsymbol{k}}$ is a given vector, then a point $P = (x, y, z)$ satisfies the vector equation $\overrightarrow{QP} \cdot \overrightarrow{B} = 2$ if and only if the variables x, y and z satisfy the equation

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 2.$$

Since the coefficients of x, y and z in this equation are a, b and c, this is an equation of a plane with normal vector equal to $\overrightarrow{B}$. Moreover, since the right hand side of the equation is not zero, the point Q can not lie on this plane. Therefore the correct choice is (E).

Solution of problem 1.8: **(a)** The equilibria for this model are solutions of the quadratic equation $12P - P^2 - H = 0$ or equivalently $P^2 - 12P + H = 0$. The discriminant of this equation is

$$(-12)^2 - 4H = 144 - 4H = 4(36 - H).$$

Therefore the quadratic equation will have two solutions only when $H < 36$.

Suppose $H < 36$. From the quadratic formula we see that the equilibria are given by

$$P_1 = 6 - \sqrt{36 - H} \quad \text{and} \quad P_2 = 6 + \sqrt{36 - H}$$

and so

$$\frac{dP}{dt} = -(P - P_1)(P - P_2).$$

This implies that $dP/dt < 0$ for $P < P_1$ and $P > P_2$ and $dP/dt > 0$ for $P_1 < P < P_2$. In particular P is decreasing when $P < P_1$, P increases when $P_1 < P < P_2$, and P decreases again when $P > P_2$. This shows that both $P = P_1$ is an unstable equilibrium and $P = P_2$ is a stable equilibrium.

(b) In order to have a single equilibrium, we must choose H so that the quadratic equation $-P^2 + 12P - H = 0$ has a unique solution. This means that the discriminant of this equation ought to be equal to zero, i.e. we ought have $4(36 - H) = 0$. Thus the special value of H is $H = 36$. In this case, the differential equation becomes

$$\frac{dP}{dt} = 12P - P^2 - 36 = -(P - 6)^2.$$

The unique equilibrium is at $P = 6$.

If $H > 36$ the equation has no equilibria, and moreover we have that

$$\frac{dP}{dt} = 12P - P^2 - H = -(P - 6)^2 + (36 - H)$$

is always negative. This shows that if we harvest the trout at rate higher than the critical harvesting rate $H = 36$, then the population will steadily decrease and we will eventually empty the pond completely. In contrast, if we harvest at a rate smaller than the critical harvesting rate $H = 36$ and if we start with enough trout in the pond, then we will always have enough fish to harvest .

Solution of problem 1.9: (a) From the parametric equation $\vec{r}(t) = (1 + 2t)\widehat{\boldsymbol{\imath}} + (1 + 3t)\widehat{\boldsymbol{\jmath}} + (1 + 4t)\widehat{\boldsymbol{k}}$ we can extract a direction vector for the line. It is the vector $\overrightarrow{v}$ whose components are given by the coefficients of the parameter t in the parametric equation. Thus $\overrightarrow{v} = 2\widehat{\boldsymbol{\imath}} + 3\widehat{\boldsymbol{\jmath}} + 4\widehat{\boldsymbol{k}}$. Also, from the equation $2x + 3y - 4z = 9$ of the plane we can extract a normal vector $\overrightarrow{n}$ to the plane. It is the vector whose components are the coefficients of the equation of the plane. Thus $\overrightarrow{n} = 2\widehat{\boldsymbol{\imath}} + 3\widehat{\boldsymbol{\jmath}} - 4\widehat{\boldsymbol{k}}$.

The line and the plane will be perpendicular when $\overrightarrow{v}$ is parallel to $\overrightarrow{n}$, that is when $\overrightarrow{v}$ is proportional to $\overrightarrow{n}$. But if $\overrightarrow{v} = c \cdot \overrightarrow{n}$ for some constant c, then we will have $2 = 2c$, $3 = 3c$, and $4 = -4c$. From he first equation we get $c = 1$ but from the last equation we have $c = -1$. This is a contradiction. Therefore the line and the plane are not perpendicular, and so (a) is False.

(b) The equation $x^2 = z^2$ depends only on two variables so it describes a cylinder with a base in the xz-plane. Hence $x^2 = z^2$ can not be an ellipsoid and (b) is False.

(c) If the length of $\vec{a} \times \vec{b}$ is zero, then the vector $\vec{a} \times \vec{b}$ must be the zero vector. This can happen either when one of $\vec{a}$ or $\vec{b}$ is the zero vector, or when $\vec{a}$ is parallel to $\vec{b}$. For instance if $\overrightarrow{a}$ is any vector and $\overrightarrow{b} = \overrightarrow{a}$ we will have $\vec{a} \times \vec{b} = \vec{a} \times \vec{a} = \overrightarrow{0}$. Hence (c) is False.

Solution of problem 1.10: (a) The unit normal vector is given by

$$\overrightarrow{N} = \frac{\frac{d\overrightarrow{T}}{dt}}{\left|\frac{d\overrightarrow{T}}{dt}\right|}$$

where $\overrightarrow{T}$ is the unit tangent vector. To compute $\overrightarrow{T}$ we compute the velocity vector $d\overrightarrow{r}/dt = \langle 2\cos(2t), -2\sin(2t), 1\rangle$ and normalize

$$\begin{aligned}\overrightarrow{T} &= \frac{1}{|d\overrightarrow{r}/dt|} d\overrightarrow{r}/dt \\ &= \frac{1}{\sqrt{4\cos^2(2t) + 4\sin^2(2t) + 1}} \langle 2\cos(2t), -2\sin(2t), 1\rangle \\ &= \left\langle \frac{2}{3}\cos(2t), -\frac{2}{3}\sin(2t), \frac{1}{3} \right\rangle.\end{aligned}$$

Hence

$$\frac{d\overrightarrow{T}}{dt} = \left\langle -\frac{4}{3}\sin(2t), -\frac{4}{3}\cos(2t), 0 \right\rangle,$$

and so

$$\overrightarrow{N} = \frac{\frac{d\overrightarrow{T}}{dt}}{\left|\frac{d\overrightarrow{T}}{dt}\right|} = \frac{3}{4}\left\langle -\frac{4}{3}\sin(2t), -\frac{4}{3}\cos(2t), 0 \right\rangle = \langle -\sin(2t), -\cos(2t), 0\rangle .$$

This is a horizontal vector pointing radially towards the z-axis. So (a) is true.

(b) From the formula for $\overrightarrow{T}$ above we see that the z component of $\overrightarrow{T}$ is constant and equal to 1/3. So (b) is true.

(c) The unit binormal vector is given by

$$\begin{aligned} \overrightarrow{B} &= \overrightarrow{T} \times \overrightarrow{N} \\ &= \det \begin{pmatrix} \widehat{\boldsymbol{\imath}} & \widehat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ \frac{2}{3}\cos(2t) & -\frac{2}{3}\sin(2t) & \frac{1}{3} \\ -\sin(2t) & -\cos(2t) & 0 \end{pmatrix} \\ &= \left\langle \frac{1}{3}\cos(2t), -\frac{1}{3}\sin(2t), -\frac{2}{3} \right\rangle . \end{aligned}$$

This vector has non-trivial x and y components so (c) is false.
