

Solutions to Problem Set #8 (advanced game theory)

Q 1. *Mathematics and Politics*, Chapter 7, page 184, problem 14. Hint: either b or c is the second largest entry. Consider the two cases separately.

Suppose we have a 2×2 zero-sum game:

	C	N
C	a	b
N	c	d

Assume that a is the largest entry and assume that d is *not* the second-largest entry. Show that either Row or Column has a dominant strategy. (Assume, for simplicity, that the numbers a , b , c , and d are distinct.)

Answer If d is not the second-largest entry, then either b or c is. If b is the second-largest, then $a > c$ and $b > d$, so that **Row has a dominant strategy of C**.

If c is the second-largest, then $a > b$ and $c > d$. Now since this is a zero-sum game, the utilities to Column are

	C	N
C	$-a$	$-b$
N	$-c$	$-d$

So $-a < -b$ and $-c < -d$, and thus **Column has a dominant strategy of N**.

Q 2. Suppose the utilities to Row of a game are

	C	N
C	2	-1
N	0	1

- a.** If Row knew that Column would pick C with a probability of 100%, what should Row's optimal strategy be? What utility would Row expect from this strategy?

Answer Row should **always pick C**, which would then result in an **expected utility of 2**.

- If Row knew that Column would pick C with a probability of 25%, what should Row's optimal strategy be? What utility would Row expect from this strategy?

Answer If Row knows Column's strategy, then Row should either always pick C or always pick N.

If Row picks C, then Row ends up in CC with probability 25% and in CN with probability 75%. The expected utility is $2 \cdot 0.25 - 1 \cdot 0.75 = -0.25$.

If Row picks N, then the expected utility is $0 \cdot 0.25 + 1 \cdot 0.75 = 0.75$.

Thus Row should **always pick N**, which results in an **expected utility of 0.75**.

Suppose we model the current conflict in Ukraine between supporters of Yushchenko and Yanukovych as a game. Let's say Yanukovych (who is already in power, and who won the disputed election) can either call for new elections or simply claim power. Let's say Yushchenko can either attempt to gain power through protesting, or concede defeat.

The utilities of the outcomes are:

- Yan. calls for new elections, Yush. concedes defeat: (+2 for Yan., -2 for Yush.)
- Yan. claims power, Yush. concedes defeat: (+6 for Yan., -10 for Yush.)
- Yan. calls for new elections, Yush. protests: (-2 for Yan., +5 for Yush.)
- Yan. claims power, Yush. protests: (-4 for Yan., +4 for Yush.)

a. Do either of the sides have a dominant strategy? If so, what?

Answer Yanukovych does not have a dominant strategy, since he prefers claiming power if Yushchenko concedes defeat, but calling for new elections if Yushchenko protests.

Yushchenko has a dominant strategy of protesting, since the protesting outcomes give him positive utility no matter what Yanukovych does, while the conceding outcomes give him negative utility no matter what Yanukovych does.

b. Are there any Nash equilibria? If so, what?

Since one side has a dominant strategy, **there is one Nash equilibrium at (-2,5)**, where Yan. calls for new elections and Yushchenko protests.

c. If Yanukovych doesn't know anything about Yushchenko's preferences, what should his optimal strategy be?

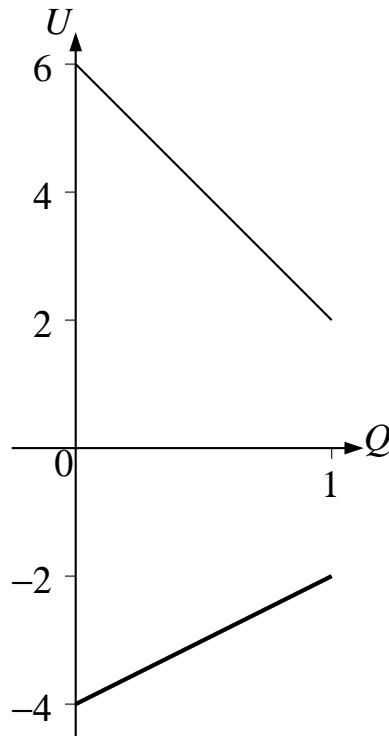
Since he doesn't have a dominant strategy, he must use a mixed strategy. Let's say he will call elections with probability Q , and claim power with probability $1 - Q$.

If Yushchenko always concedes defeat, Yanukovych gets an expected utility of

$$U = 2Q + 6(1 - Q) = 6 - 4Q.$$

If Yushchenko always protests, Yanukovych gets an expected utility of

$$U = -2Q - 4(1 - Q) = -4 + 2Q.$$



The graphs of these two lines is shown above. The lower one is the one Yanukovych should use to make his decision. The highest point on that line is when $Q = 1$. So Yanukovych should **always call elections**.

- d. If Yanukovych knows Yushchenko's preferences, what should his optimal strategy be?

Since Yan. knows that Yush. will always follow Yush.'s dominant strategy, he should **call new elections**.

- e. If Yushchenko doesn't know anything about Yanukovych's preferences, what should his optimal strategy be?

Answer Since he has a dominant strategy of protesting, he should **protest** no matter what Yanukovych might do.

- f. If Yushchenko knows Yanukovych's preferences, what should his optimal strategy be?

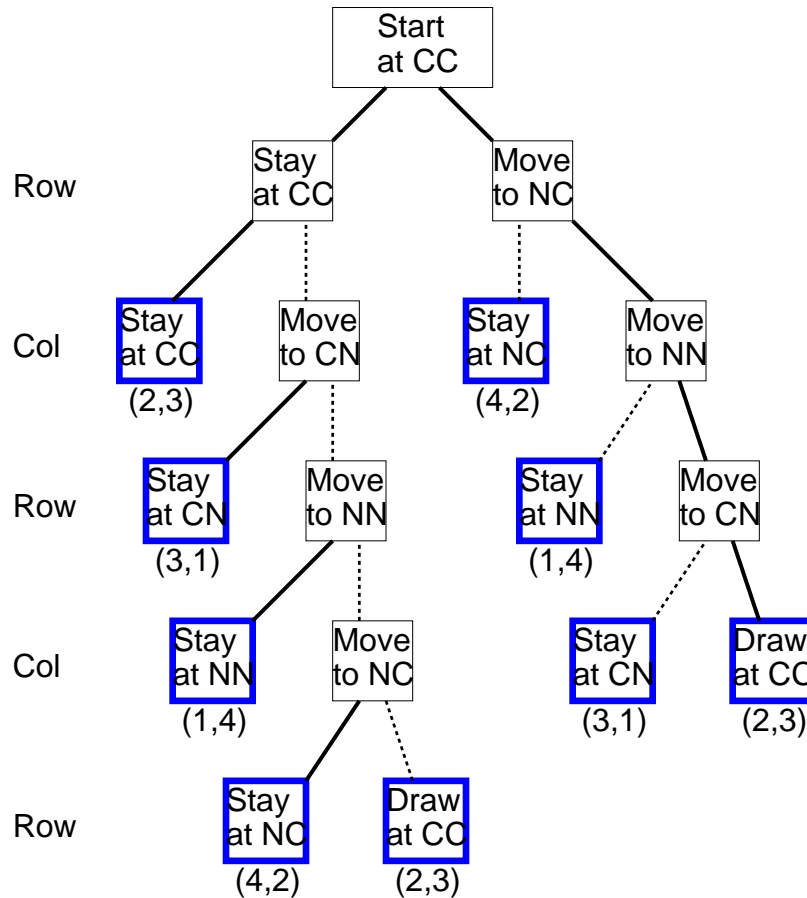
Answer Again he should follow his dominant strategy and **always protest**.

Q 4. Consider the following game. Starting from CC, with Row going

first, find the Theory of Moves prediction. Use the “draw” criterion as in class, i.e. if the players end up back at CC, the game ends.

	C	N
C	(2,3)	(3,1)
N	(4,2)	(1,4)

Answer The full theory of moves decision tree is shown below.



We see that the players have two optimal strategies which will result in the same outcome: either Row stays at CC and Column stays at CC; or Row moves to NC, Column moves to NN, Row moves to CN, and Column moves back to CC for a draw. Since either way results in the same ultimate outcome of CC, the two players might as well **both stay at CC**.

Q 5. Consider this model of conflict between Rwanda and Congo. Both countries can either send troops to their border or negotiate. Rwanda

prefers that Congo does not send troops, and secondarily prefers sending troops. Congo prefers that Rwanda does not send troops; if Rwanda does send troops, Congo prefers to also send troops, but if Rwanda does not, Congo prefers to negotiate.

- a. Write the matrix of a 2×2 ordinal game representing this situation. Determine the Nash equilibrium.

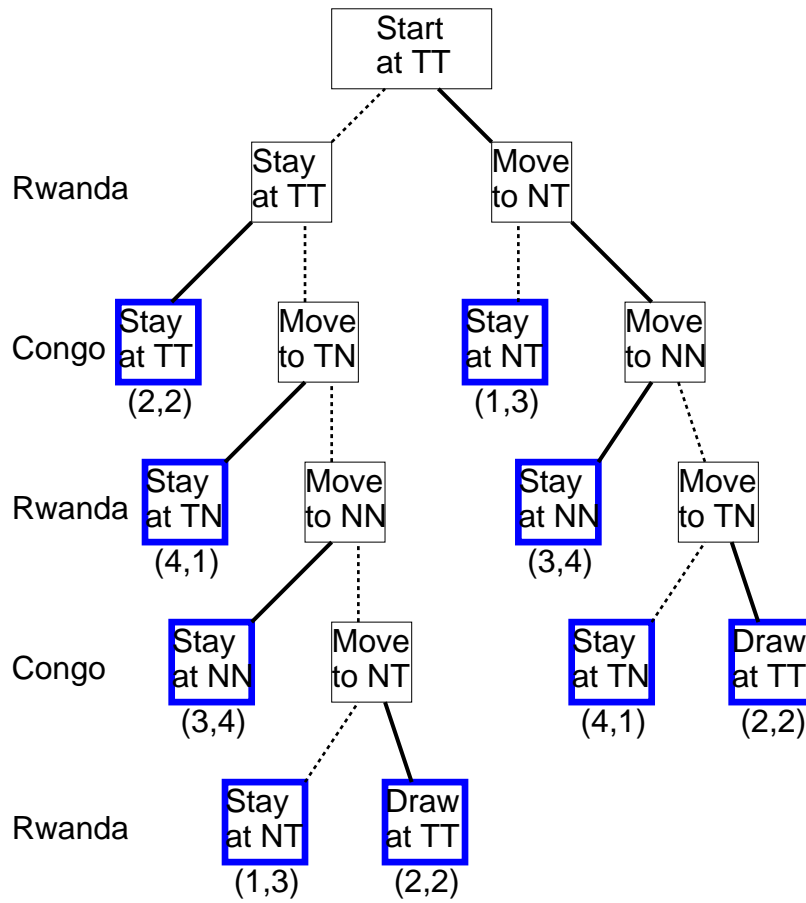
Answer With Rwanda as Row and Congo as Column, t T representing troops and N representing negotiating, the matrix is

	N	T
N	(3,4)	(1,3)
T	(4,1)	(2,2)

Rwanda has a dominant strategy of sending troops, and therefore there is one Nash equilibrium: **both sides send troops**.

- b. Starting from the Nash equilibrium, with Rwanda moving first, use the Theory of Moves to determine whether the two sides will move away from the Nash equilibrium.

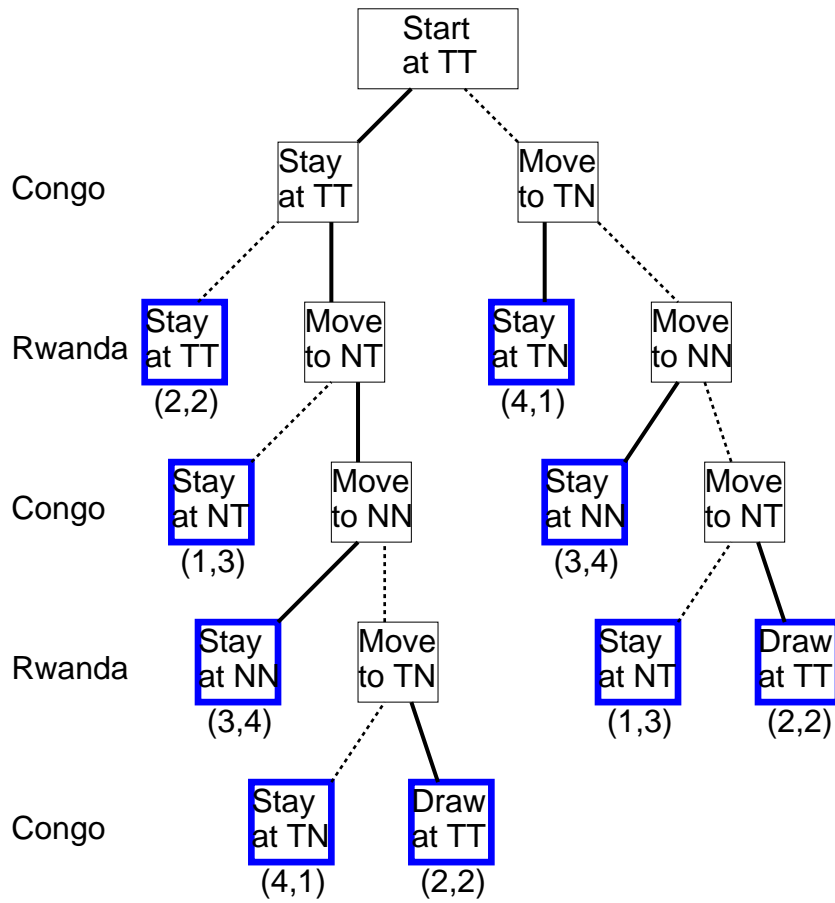
Answer Here is the decision tree:



We see that Rwanda's optimal strategy is to move *away* from its dominant strategy: Rwanda will change to negotiating, Congo will also change to negotiating, and Rwanda will stay there.

- c. Do the same thing with Congo moving first.

Answer Here is the decision tree:



We see that Congo's optimal strategy is to stay at the Nash equilibrium, knowing that Rwanda will tend to move away (as in part b). Then Congo will stay with troops, Rwanda will change to negotiating, Congo will also change to negotiating, and Rwanda will stay. If Congo were to move to negotiating, it would be giving Rwanda its highest preference, and therefore Rwanda would stay there.