Solutions to Problem Set #7 (game theory)

Q 1. *Mathematics and Politics*, pg. 19, problem 15.

According to O'Neill's theorem, what should the opening bid be in rational play of the dollar auction for the case where stakes are two dollars, bankroll twenty dollars, and units quarters?

Answer Here the stakes are $s = \$2.00/\$0.25 = 8$ units, while the bankroll is $b = \$20.00/\$0.25 = 80$ units. So we subtract off multiples of $(s-1) = 7$ from $b = 80$ until we get zero. Clearly $b - 11(s-1) = 80 - 77 = 3$, and subtracting off 7 any more times will give us something less than zero. So the optimal bid is 3 units, or \$0.75.

Q 2. *Mathematics and Politics*, pp. 35–42: 1, 4, 8, 9, 10, 13.

1. Suppose Row ranks the four possible outcomes, from best to worst, in a 2×2 ordinal game as CN, CC, NC, NN and Column ranks the four, again from best to worst, as CC, NN, NC, CN.

Set up the 2×2 matrix giving Row's preference ranking, the one giving Column's preference ranking, and the one containing both.

Answer

	C	N
C	3	4
N	2	1

	C	N
C	4	1
N	2	3

	C	N
C	(3,4)	(4,1)
N	(2,2)	(1,3)

4. In the following 2×2 ordinal game:

	C	N
C	(2,3)	(3,1)
N	(4,2)	(1,4)

(a) Show that C is *not* a dominant strategy for Row.

Answer If Column picks C, Row is better off picking N.

(b) Show that N is not a dominant strategy for Row.

Answer If Column picks N, Row is better off picking C.

(c) Show that C is not a dominant strategy for Column.

Answer If Row picks C, Column is better off picking N.

(d) Show that N is not a dominant strategy for Column.

Answer If Row picks N, Column is better off picking C.

8. Does Column have a dominant strategy in the following 2×3 game where Column has three choices: C, N, and V? Each ranks the six possible outcomes from 6 (best) to 1 (worst).

	C	N	V
C	(5,4)	(3,5)	(2,6)
N	(6,1)	(4,2)	(1,3)

Answer Yes. If Row picks C, Column should pick V, since 6 is higher than 4 and 5. If Row picks N, Column should again pick V, since 3 is higher than 1 and 2. So V is the dominant strategy for Column.

9. Suppose that CC is a (4,4) outcome in a 2×2 ordinal game. Does this guarantee that C is a dominant strategy for both Row and Column? (Either explain why it does, or find a 2×2 ordinal game showing that it need not.)

Answer No. Here is an example:

	C	N
C	(4,4)	(2,1)
N	(1,2)	(3,3)

If Row picks N, Column should pick N as well. Similarly, if Column picks N, Row should pick N. So C cannot be a dominant strategy for either one. (This is slightly more than was asked, though.)

10. In the following game:

	C	N
C	(2,3)	(4,2)
N	(1,1)	(3,4)

(a) Show that (2,3) is a Nash equilibrium.

Answer If Row moves, Row will go to a preference of 1, which is worse. If Column moves, Column will go to a preference of 2, which is worse.

(b) Show that (4,2) is not a Nash equilibrium.

Answer If Column moves, Column will go to a preference of 3, which is better.

(c) Is (3,4) a Nash equilibrium? (Why or why not?)

Answer No. If Row moves, Row will go to a preference of 4, which is better.

(d) Is (1,1) a Nash equilibrium? (Why or why not?)

Answer No. If Row moves, Row will get a higher preference of 2. If Column moves, Column will get a higher preference of 4.

13. Find all Nash equilibria in the following 3×3 game:

	C	N	V
C	(1,9)	(4,2)	(7,7)
N	(3,4)	(9,3)	(5,1)
V	(6,5)	(2,6)	(8,8)

Answer (1,9) is not, since Row wants to move to (6,5). (4,2) is not, since both Row and Column want to move. (7,7) is not, since Column wants to move to (1,9). (3,4) is not since Row wants to move to (6,5). (9,3) is not, since Column wants to move to (3,4). (5,1) is not, since Column wants to move to (3,4). (6,5) is not, since Column wants to move to (8,8). (2,6) is not, since both Row and Column want to move.

Only (8,8) is a Nash equilibrium, since Column thinks 8 is better than 5 and 6, while Row thinks 8 is better than 7 and 5.

Q 3. How many different 2×2 ordinal games are there? Explain your answer *without* writing them all out.

Answer We are asking how many ways there are to fill up a 2×2 matrix:

	C	N
C	(,)	(,)
N	(,)	(,)

First, ask how many ways there are to fill up Row's preferences. We must use all the numbers from 1 to 4, but we can put them in any order. The number of different ways to order the numbers $\{1, 2, 3, 4\}$ is $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$.

For every way we fill up Row's preferences, we have another $4! = 24$ ways of filling up Column's preferences.

By the multiplication principle, the total number of ways to fill up both player's preferences is

$$24 \cdot 24 = 576.$$