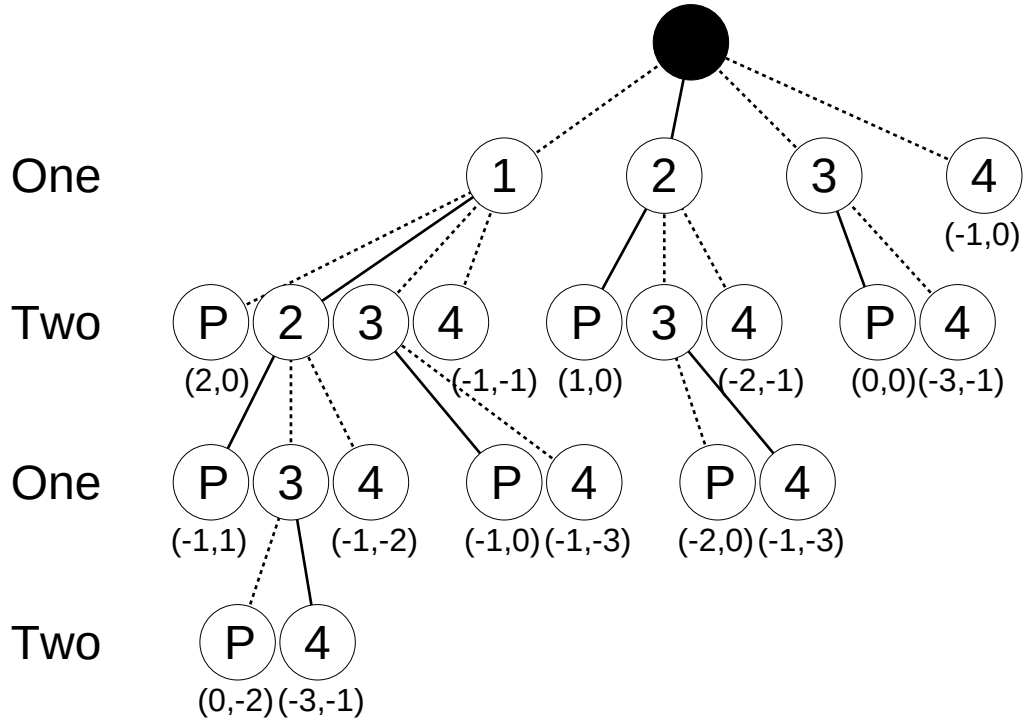
Solutions to Problem Set #6 (decision trees)

Q 1. *Mathematics and Politics*, pg. 17, problem 1.

Do the game-tree analysis for the standard dollar auction with the conservative convention for the case where $s = 3$ and $b = 4$. (The tree will have fifteen terminal nodes, and the analysis will show that the optimal opening bid is *not* 1 unit as was the case for $s = 3$ and $b = 3$.)

Answer The decision tree is shown below.



The tree tells us that the optimal strategy is for player one to bid 2; player two will then pass.

Q 2. Suppose you are trying to explain the optimal strategy for bidding in the dollar auction with stakes $s = 3$, assuming that both players use the conservative convention. You point out that if the bankroll is $b = 3$, then the optimal strategy is to bid 1 and the other player will pass; however if the bankroll is $b = 4$, the optimal strategy is different (see problem 1 above).

Your listener does not understand the difference. "Surely," says he, "if the optimal strategy involves never actually going past 3, then the bankroll doesnt affect the outcome." Explain to him why this is not the case, without using the decision tree (he gets lost in the symbols) or O'Neill's Theorem (he doesn't trust this O'Neill character anyway).

In other words, just explain in plain English where the distinction arises.

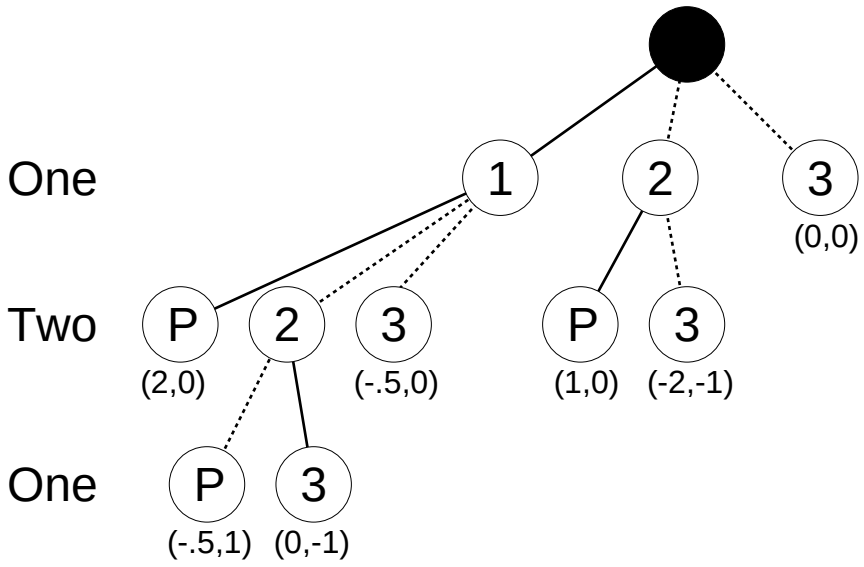
Answer If the bankroll is only 3, then when player one bids 1, player two has no motivation to bid, since bidding 2 will force player one to bid 3 and then player two has lost it all.

However, if the bankroll is increased, and player one bids 1, player two will want to bid 2; then player one cannot stop the bidding at 3 and is faced with either bidding 4 or passing. Either way player one loses 1, so player one will pass using the conservative convention. Thus if player one bids 1, player two will win.

Q 3. *Mathematics and Politics*, pg. 18, problem 4a. (The notation is defined in the middle of page 18, and refers to the definitions on the bottom of 17 and top of 18.)

Do the game-tree analysis of the Ai-Bii-Ci dollar auction for the case where $s = 3$ and $b = 3$. (Use the conservative convention and the rule that the second-highest bidder pays half of what he or she bid.)

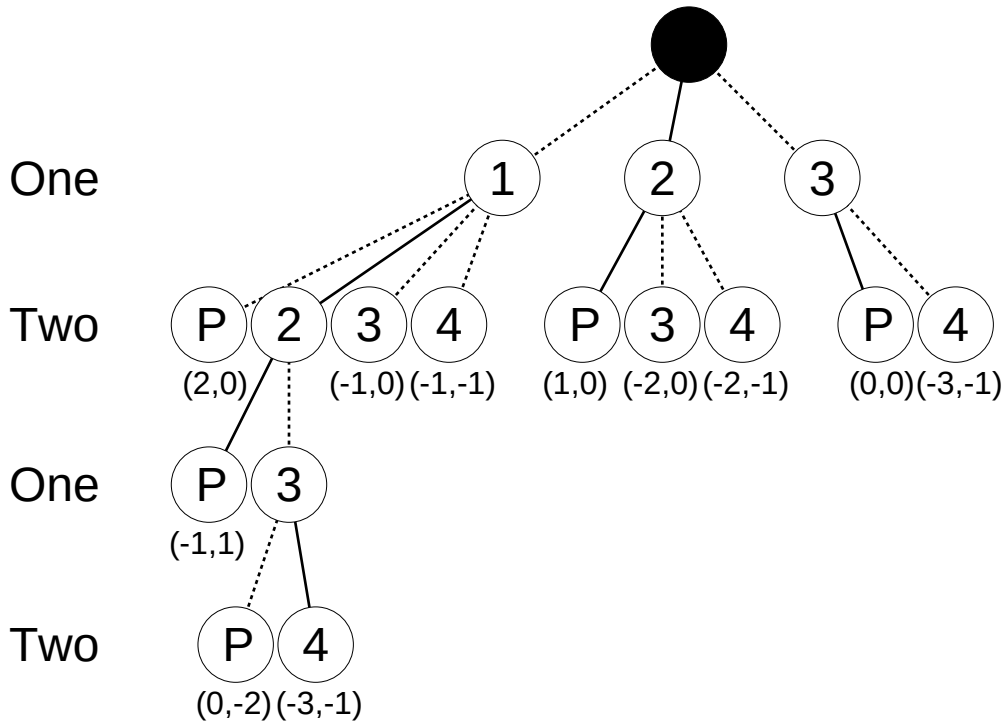
Answer The decision tree is shown below.



The tree tells us that the optimal strategy is for player one to bid 1; player two will then pass.

Q 4. Suppose you are playing the dollar auction with stakes $s = 3$. Your bankroll is $b = 3$; however, player twos bankroll is $b = 4$. In other words, you can bid at most 3 but player two can bid up to 4. Draw the decision tree. What is your optimal strategy?

Answer The decision tree is shown below.



Your optimal strategy is to bid 2; player two will then pass.