

# Logic and voting systems: Homework

Mathematics 170

due Friday, October 1

1. Negate the sentence: “For every vote Senator Specter gets, he has to spend \$10 or one hour of volunteer time.”
2. Let  $P(x, y) = “x \text{ gives a contribution to } y,”$   $x = “\text{corporation},”$  and  $y = “\text{politician}.”$ 
  - (a) Write the statement  $\exists x \forall y P(x, y)$  in English.
  - (b) Write the statement  $\forall y \exists x P(x, y)$  in English.
  - (c) Which statement implies the other one?
  - (d) Write the statement “Every corporation gives a contribution to some politician” symbolically.
  - (e) Write the statement “A politician got a contribution from a corporation” symbolically.
3. Consider the following axioms. If the IMF gives a country a loan, then it will impose structural adjustment. If a country has a structural adjustment program and provides welfare benefits, then the IMF did not give it a loan. If a country does not provide welfare benefits, then it experiences high poverty.  
Prove that if a country does not experience high poverty, then the IMF did not give it a loan. Use either proof by contradiction or a direct proof.
4. Chapter 5, problems 3 and 9.
5. Chapter 5, problem 11. Hint: The easiest way to do this is to prove two parts separately. For the first, assume some alternative is a Condorcet winner, and show that that alternative wins sequential pairing with any agenda.  
For the second, assume some alternative is not a Condorcet winner. Show that there is at least one agenda that will make that person lose a sequential pairwise vote. (Construct one.)