Solutions to Problem Set #1 (Introduction to Logic)

Q 1. Write the negation of the following sentence: "Donate to a campaign, or knock on doors and stuff envelopes."

Answer Symbolically, if we write $P =$ "donate to a campaign," $Q =$ "knock on doors," and $R =$ "stuff envelopes," then the given statement is

$$P \textbf{ or } (Q \textbf{ and } R).$$

The negation is

$$\begin{aligned} &\textbf{not } \big[P \textbf{ or } (Q \textbf{ and } R)\big] && \text{(negation)} \\ &\textbf{not } P \textbf{ and not } (Q \textbf{ and } R) && \text{(DeMorgan law)} \\ &\textbf{not } P \textbf{ and } (\textbf{not } Q \textbf{ or not } R) && \text{(DeMorgan law)} \end{aligned}$$

In English, the negation is thus: "Don't donate to a campaign, and either don't knock on doors or don't stuff envelopes."

Notice that the location of the comma is important: it corresponds to the placement of parentheses in the symbolic notation. Putting the comma in a different place than is here changes the meaning of the sentence.

Also, a symbolic argument is not necessary; for this problem, it's OK to just write the answer, as long as you can do it correctly.

Q 2. Write the contrapositive of the following sentence: "If the war in Iraq improves or the economy improves, Bush will win the election."

Answer Again, we write $P =$ "the war in Iraq improves," $Q =$ "the economy improves," and $R =$ "Bush wins the election." The statement is:

$$\textbf{If } (P \textbf{ or } Q), \textbf{ then } R.$$

The contrapositive is then

$$\begin{aligned} &\textbf{If not } R, \textbf{ then not } (P \textbf{ or } Q). \\ &\textbf{If not } R, \textbf{ then } (\textbf{not } P \textbf{ and not } Q). \end{aligned}$$

(We used the DeMorgan law: the negation of 'and' is 'or' of the negations.)

In English, the contrapositive is: "If Bush does not win the election, then the war in Iraq didn't improve and the economy didn't improve."

Q 3. Construct a truth table for the statement "NOT (P AND Q) OR R."

Answer The truth table will have eight rows. We will construct an intermediate column to give 'P **and** Q' and '**not** (P **and** Q),' just to simplify the work (it's not essential). The sixth column is the 'or' of the third and fifth columns.

P	Q	R	P **and** Q	**not** (P **and** Q)	**not** (P **and** Q) **or** R
T	T	T	T	F	**T**
T	T	F	T	F	**F**
T	F	T	F	T	**T**
T	F	F	F	T	**T**
F	T	T	F	T	**T**
F	T	F	F	T	**T**
F	F	T	F	T	**T**
F	F	F	F	T	**T**

Q 4. Show that the statements "P OR (Q AND R)" and "(P OR Q) AND (P OR R)" are equivalent, by showing that they have the same truth table.

Answer The two statements we want to compare are in the bold columns. We construct intermediate steps: the fourth column is 'and' of the second and third columns; the fifth is 'or' of the first and fourth; the sixth is 'or' of the first and second; the seventh is 'or' of the first and third; and the eighth is 'and' of the sixth and seventh.

P	Q	R	Q **and** R	P **or** (Q **and** R)	P **or** Q	P **or** R	(P **or** Q) **and** (P **or** R)
T	T	T	T	**T**	T	T	**T**
T	T	F	F	**T**	T	T	**T**
T	F	T	F	**T**	T	T	**T**
T	F	F	F	**T**	T	T	**T**
F	T	T	T	**T**	T	T	**T**
F	T	F	F	**F**	T	F	**F**
F	F	T	F	**F**	F	T	**F**
F	F	F	F	**F**	F	F	**F**

Since the two bold columns are exactly the same, the two operations are equivalent.

Q 5. Write the truth table for the statements "IF P, THEN (Q AND R)," "IF (P AND Q), THEN R," and "IF P, THEN (IF Q THEN R)." Are any of these equivalent?

Answer Again we break up the statements into simpler parts. Recall that "If P, then Q" is true unless P is true and Q is false.

P	Q	R	Q **and** R	**If** P **then** (Q **and** R)	P **and** Q	**If** (P **and** Q) **then** R	**If** Q **then** R	**If** P **then** (**if** Q **then** R)
T	T	T	T	**T**	T	**T**	T	**T**
T	T	F	F	**F**	T	**F**	F	**F**
T	F	T	F	**F**	F	**T**	T	**T**
T	F	F	F	**F**	F	**T**	T	**T**
F	T	T	T	**T**	F	**T**	T	**T**
F	T	F	F	**T**	F	**T**	F	**T**
F	F	T	F	**T**	F	**T**	T	**T**
F	F	F	F	**T**	F	**T**	T	**T**

Since the seventh and ninth columns are the same, we see that "If (P and Q), then R" is equivalent to "If P, then (if Q, then R)."

Q 6. How many different truth tables could be constructed for an operation between two statements P and Q? Hint: for each truth value of P and of Q, one can choose either true or false. How many possibilities are there?

Answer A generic truth table for an operation $P \otimes Q$ looks like

P	Q	$P \otimes Q$
T	T	
T	F	
F	T	
F	F	

Now there are four slots, and we can fill in each one of them with two possibilities: either 'T' or 'F'. Thus we have two choices for the first row, two for the second, two for the third, and two for the fourth. Since none of these choices depends on the other, there are

$$2 \times 2 \times 2 \times 2 = 16$$

possible ways to fill in the truth table.

Q 7. Write a true if-then statement whose converse is false. Write one whose converse is true.

Answer

(a) (False converse): "If I am the President, my salary is over $100,000 per year." The converse is, "If my salary is over $100,000 per year, then I am the President," which is false.

(b) (True converse): "If $x = 2$, then $x + 3 = 5$." The converse is, "If $x + 3 = 5$, then $x = 2$," which is also true.

Q 8. A certain bill came up for a vote in the Senate. If Trent Lott voted for it, Ted Kennedy promised not to vote for it. Jim Jeffords said hed vote for it only if Lott voted for it. If Kennedy voted for it, then either Jeffords or Lott voted against it.

(a) Let J, K, and L denote the statements "Jeffords voted yes," "Kennedy voted yes," and "Lott voted yes," respectively. Assuming everyone kept their promises, write the assumptions above using IF...THEN, NOT, AND, and OR.

(b) If Kennedy voted for the bill, determine how Jeffords and Lott voted.

Answer The assumptions symbolically are

$$\textbf{If } L, \textbf{ then not } K$$
$$\textbf{If } J, \textbf{ then } L$$
$$\textbf{If } K, \textbf{ then } (\textbf{not } J \textbf{ or not } L)$$

Now the contrapositives of the first and second statements are

$$\textbf{If } K, \textbf{ then not } L$$
$$\textbf{If not } L, \textbf{ then not } J$$

So K implies **not** L, and then **not** L implies **not** J.

Thus Jeffords and Lott both voted against the bill.