TEACHING STATEMENT

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As a teacher of mathematics, I want to encourage students to become independent thinkers. Some of my students will go on to use advanced mathematics in their daily lives, as I have. But all of them will have to deal with complex problems that do not closely resemble those that they have seen before. To encourage students' thinking, I provide ample feedback, concrete examples, and illustrate connections between different subfields of mathematics. I also stress the importance of communicating mathematics properly while acknowledging that the shortest path to a proper solution may go through the jungles of heuristic and conjecture.

I believe in the importance of giving students large amounts of feedback on their work. First, I prepare detailed solutions to homework assignments, which I distribute to students, usually electronically. When there is more than one substantially different way to solve a problem I often include alternate approaches – including ones that I may not have thought of that were suggested by students. Students have commented that they find these solutions quite useful when studying for exams. I also do my best to grade homework quickly and thoroughly. When time permits, I make sure to spend class time on sketching correct solutions to problems, in particular to those problems that gave a large number of students difficulty. I keep in mind that homework exists both to evaluate students and to help them learn. Therefore when a student makes an error, I make sure to tell them what the mistake is and not just subtract points. I return the homework quickly enough that perhaps the student remembers what they were thinking when they wrote that line.

I like to show students concrete examples. In more advanced classes there is often too much of a tendency towards abstraction. The fundamental examples are existence theorems – for example, the Stone-Weierstrass theorem, which tells us that certain algebras of functions on a metric space are dense in the space of all continuous real-valued functions on a metric space. Upon teaching this I learned that the word "dense" was unfamiliar to some of my students. But this theorem just means that arbitrary continuous functions can be approximated arbitrarily closely by functions in a certain algebra; some examples with which students are familiar are polynomials and Fourier series. I find that illustrating these concrete examples gives students a pathway by which to understand the abstraction (and I show computer graphics, which gets the attention of students whose minds may have wandered). At the same time, the fact that two pieces of mathematics which appear to be disjoint are at a deeper level the same is a powerful lesson.

I want to show my students that mathematics is an interconnected whole. The nature of the mathematics curriculum compartmentalizes material; students often assume that what they learned in one class is not necessary for another one. The technique of showing two distinct proofs of the same result is not just for advanced classes. In the course "Ideas in Mathematics", one theme I returned to multiple times was the Platonic solids. There are five of them. Euclid showed this by a geometrical argument, and there is also a graph-theoretical argument that requires counting vertices, edges, and faces. Surely one proof is logically enough. But I want students to really understand *why* something is true; some students better understand one argument, some the other. My goal is to do more than show students the logical skeleton of the subject, which they can read about anyway. I want to put flesh on those bones. Ideally I want the student to believe that *they* "could have thought of that".

I believe it is important that students write complete and correct solutions. In classes and now in my research I have often had the experience of believing I understood the solution to a problem until sitting down to write; at that time I often realize that my understanding is incomplete. I find that many of my students have the same experience. In one calculus class I was involved in we deliberately assigned only a few problems a week to be handed in, and insisted that students write the solutions up well, using complete sentences and proper English. The students resisted at first, but over the course of the semester I saw definite improvement in their ability to write mathematics. I hope that this translated into improved ability to think about mathematics.

While written solutions should be complete and correct, I encourage students to be incorrect in the classroom. This does not mean that I want them to say things that are clearly *wrong*. But I do try to encourage them to share partial solutions, to suggest ideas that may or may not work, and generally to not be afraid of conjecture. This is part of a more general attempt to show that mathematics is a creative enterprise. It is difficult for students to see this, because they feel they are not creating anything new but merely walking in the footprints of giants. To encourage this, I try to create a lighthearted environment in the classroom. I do my best to be approachable to students and build a rapport with them, so that they become active learners instead of sitting back and hoping to learn something by osmosis.

We mathematicians know that mathematics is one of the great achievements of the human mind. We know the thrill of discovering a new piece of mathematics for the first time. Not all our students are suited for the mathematical life. But I want them to see this nonetheless. I want them to know that mathematics is interesting and vital. Most of all, I want to get them thinking.