Final Examination

Directions: Answer all 5 questions. Time: One hour. You may use one sheet of A4 paper with notes on *one* side. Try to communicate your ideas clearly.

1. Let $f \in C^2(\mathbb{R})$ be a 2π periodic function, so f and its first two derivatives are 2π periodic. Say $f(x) = \sum_k c_k e^{ikx}$ is its Fourier series.

   a) Show there is a constant m so that $|c_k| \le \dfrac{m}{1+k^2}$.

   b) Show that the Fourier series converges uniformly.

2. Let $u(x,t)$ be a solution of $u_{tt} + b(x,t)u_t = u_{xx}$ for $0 < x < L$. Assume u satisfies the initial conditions $u(x,0) = 0$ and $u_t(x,0) = 0$ and boundary conditions $u(0,t) = u(L,t) = 0$.

   a) If $b(x,t) \ge 0$, show that $u(x,t) = 0$ for all $t > 0$.

   b) If $|b(x,t)| \le M$ for some constant M show that $u(x,t) = 0$ for all $t > 0$.

3. Let $\Omega \subset \mathbb{R}^2$ be a bounded open set and $u(x,t)$ a solution of $u_t = \Delta u$ in Ω with $u(x,t) = f(x)$ for $x \in \partial\Omega$. Also, let $v(x)$ satisfy $\Delta v = 0$ in Ω with $v(x) = f(x)$ on $\partial\Omega$. Show that, in an appropriate sense,

$$\lim_{t\to\infty} u(x,t) = v(x).$$

4. Let $\Omega \in \mathbb{R}^3$ be a bounded open set. Assume $Lu := -\Delta u + c(x)u \ge 0$, where $c(x) > 0$ is a continuous function.

   a) Show that u cannot assume a negative minimum at any point of Ω.

   b) If u and v satisfy $Lu = f$ and $Lv = g$, respectively, in Ω with $f > g$ in Ω and $u = v$ on $\partial\Omega$, what can you conclude? Proof?

5. Pick a topic (or technique) in the course that interested you and give a brief summary of it. You may include theorems, proofs, ideas, examples, special cases, etc. You don't need to be really precise, but give the main ideas – as if you were describing it to a friend at coffee.

   [Please don't jabber. First think and plan calmly. *Please do not write more than one page.*]