Homework Set 8

DUE: Tues, March 31, 2009.

The *Problem Collection* is at
`http://www.math.upenn.edu/ kazdan/609S09/hw/hw-collection.html`

1. Problem Collection p. 12 #6

2. Problem Collection p. 22 #5

3. Problem Collection p. 51 #3, 4 [deleted: does not exist)

4. Problem Collection p. 54 #12, 15

5. Problem Collection p. 55 #18

6. Problem Collection p. 56 #26

7. Problem Collection p. 57 #31

8. Some results on Infinite Products.

   a) Let $0 \le b_k$ and $b_k \ne 1$. If $\sum b_k$ converges, show that $\Pi(1-b_k)$ converges. [HINT: First do the special case where $b_1 + b_2 + \cdots < \frac{1}{2}$. The general case reduces to this by simply discarding a finite number of terms.]

   b) If $0 \le b_k < 1$ but $\sum b_k$ diverges, show that $\Pi(1-b_k)$ diverges to 0.

   c) Let $a_k \in \mathbb{C}$. Show that $\Pi(1+a_k)$ converges absolutely if and only if $\sum |a_k|$ converges.

   d) If $\Pi(1+a_k)$ converges absolutely, prove that $\Pi(1+a_k)$ converges.

9. Let f be an entire function that is never zero. Show there is an entire function g so that $f(z) = e^{g(z)}$. [HINT: . Use the entire function $h(z) := f'(z)/f(z)$ to obtain g].

Bonus Problem Let f be an entire function with the two properties:

$$a).\ \ f(x+2\pi) = f(x) \ \text{ for all } \ x \in \mathbb{R}, \qquad b).\ \ |f(z)| \le e^{c|z|} \ \text{ for some } \ c > 0.$$

Show that f has the form $f(z) = \sum_{k=-n}^{n} a_k e^{ikz}$ where $n \le c$.