

CSE 313
Midterm Examination
March 5, 2004

Question 1: { 15 pts }

Consider the following system of linear equations:

$$3w + x + 2y + z = -4$$

$$w + 4x + y + 3z = 13$$

$$5w - 2x + 3y - z = -21$$

Give the *complete* solution to this system – show your work.

Question 2: { 15 pts }

If A is a square matrix such that $(I - A)$ is nonsingular prove that:

$$A(I - A)^{-1} = (I - A)^{-1}A$$

Question 3: { 15 pts }

Suppose two matrices, A and B , are row equivalent, that is there exists a nonsingular matrix P such that $PA = B$. Answer the following questions about A and B . Explain your answer in each case.

- Do A and B have the same column space ie does: $R(A) = R(B)$
- Do A and B have the same null space ie does: $N(A) = N(B)$
- Do A and B have the same row space ie does: $R(A^T) = R(B^T)$
- Do A and B have the same left hand null space ie does:
 $N(A^T) = N(B^T)$

Question 4: { 15 pts }

Show that the following set of vectors constitute a basis for \mathbb{R}^3 .

$\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \right\}$ Compute the coordinates of the following vectors

with respect to this basis: $\left\{ \begin{pmatrix} 3 \\ 5 \\ -5 \end{pmatrix}, \begin{pmatrix} 6 \\ 2 \\ -2 \end{pmatrix} \right\}$

Question 5: { 15 pts }

If x and y are vectors such that $\|x - y\|_2 = \|x + y\|_2$ what is $x^T y$?

Question 6: { 15 pts }

If A is a square matrix prove that

$$\|A\|_F^2 = \|A^+\|_F^2 + \|A^-\|_F^2$$

where $\|A\|_F^2 = \text{tr}(A^T A)$ denotes the square of the Frobenius norm of A , $A^+ = \frac{A + A^T}{2}$ denotes the symmetric part of A and $A^- = \frac{A - A^T}{2}$ denotes the skew symmetric part of A