Math 312, Homework 1 (due Friday, Sep. 14)

Name:_____ (if you choose to use this as a coversheet)

Reading

- Read chapter 1 and section 2.1 of Bretscher.

Book problems

- Section 1.1, problems 8, 22, 35, 47

- Section 1.2, problems 9, 18, 30, 35

- Section 1.3, problems 1, 2, 3, 4, 6, 22, 24

- Section 2.1, problems 1, 2, 3, 6

Additonal problems

1. Let A and B be 2×2 matrices. Show by examples that the rank of $A + B$ could be smaller than, greater than, or equal to the rank of A.

2. Let
$$A = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}, \qquad \vec{b} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

   First, solve $A\vec{x} = \vec{b}$. Then draw two pictures: one that describes the solution as intersecting lines and another that expresses $\vec{b}$ as a linear combination of the columns of A.

3. Let
$$A = \begin{bmatrix} 1 & 1 & -1 \\ -4 & 1 & 1 \\ 1 & -5 & 3 \end{bmatrix}, \qquad \vec{b} = \begin{bmatrix} -1 \\ 9 \\ -7 \end{bmatrix}.$$

   Find scalars x_1, x_2, x_3 that give $\vec{b}$ as a linear combination of the columns of A.