Math 103: Antiderivatives and the Area Under a Curve

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Outline

Definition

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Theorem

If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is

$$F(x) + C$$

where C is an arbitrary constant.

Given $F' = f$ and $G' = g$

Function	**Particular Antiderivative**
$cf(x)$	$cF(x)$
$f(x) + g(x)$	$F(x) + G(x)$
$x^n \quad n \neq 1$	$\frac{x^{n+1}}{n+1}$
$cos(x)$	$sin(x)$
$sin(x)$	$-cos(x)$
$sec^2(x)$	$tan(x)$
$sec(x)tan(x)$	$sec(x)$

Indefinite Integral

Definition

The collection of all antiderivatives of f is called the **indefinite integral** of f with respect to x, and is denoted by

$$\int f(x)dx$$

Example

Show that for motion in a straight line with constant acceleration a, initial velocity v_0 and and initial displacement s_0, the displacement after time t is given by

$$S(t) = \frac{1}{2}at^2 + v_0 t + s_0$$

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3. If we choose the height of each rectangle to be the value of f(x) at the **midpoint** of the base interval, the estimate is an **midpoint sum**

n	L_n	U_n
10	.285	.385
20	.308	.358
30	.316	.350
50	.323	.343
100	.328	.338
1000	.333	.334

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Then we pick a point c_k in the k-th subinterval and estimate the height of the rectangle as $f(c_k)$.
Then an estimate of the area is given by the following sum

$$f(c_1) \cdot \Delta x + f(c_2) \cdot \Delta x + ... + f(c_n) \cdot \Delta x$$