Math 103: Trig Derivatives and Rate of Change Problems

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Thursday February 9, 2012

Outline

1. Review
2. Rates of change
3. Trig derivatives

Derivative Rules

1. $\frac{d}{dx}(c) = 0$
2. $\frac{d}{dx}(x^n) = nx^{n-1}$
3. $\frac{d}{dx}(cf(x)) = c\frac{d}{dx}(f(x))$
4. $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$
5. $\frac{d}{dx}[a^x] = ln(a)a^x$
6. $\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}(g(x)) + g(x)\frac{d}{dx}(f(x))$
7. $\frac{d}{dx}[\frac{f(x)}{g(x)}] = \frac{g(x)\frac{d}{dx}(f(x)) - f(x)\frac{d}{dx}(g(x))}{(g(x))^2}$

Instantaneous Velocity

Definition

If $s(t)$ is a position function defined in terms of time t, then the instantaneous velocity at time $t = a$ is given by

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ExampleSuppose a penny is dropped from the top of DRL which is 19.6 meters high. The position of the penny in terms of hight above the street is given by $s(t) = 19.6 - 4.9t^2$. At what is the velocity of the penny when it hits the ground.

Position, Velocity, Acceleration and Jerk

If the position of a body at time t is given by $s(t)$ then

1. Velocity at time t is given by $v(t) = \frac{ds}{dt}$
2. Acceleration at time t is given by $a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$
3. Jerk at time t is given by $j(t) = \frac{da}{dt} = \frac{d^2v}{dt^2} = \frac{d^3s}{dt^3}$

Harmonic Motion

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$$s(t) = -3cos(t)$$

What is the velocity and acceleration of the weight at time t?

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Lemmas:

1. $lim_{\theta \to 0} \frac{sin(\theta) - sin(0)}{\theta - 0} = lim_{\theta \to 0} \frac{sin(\theta)}{\theta} = 1$
2. $lim_{\theta \to 0} \frac{(cos(\theta) - cos(0))}{\theta - 0} = lim_{\theta \to 0} \frac{(cos(\theta) - 1)}{\theta} = 0$

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3. $sin(x+h) = sin(x)cos(h) + cos(x)sin(h)$
4. $cos(x+h) = cos(x)cos(h) - sin(x)sin(h)$

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5. $\frac{d}{dx}(cot(x)) = -(csc(x))^2$