Math 103: Derivatives and Derivative Rules

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Outline

Derivatives from Limits

1. Secant lines to functions.
2. Tangent lines to functions.
3. Finding the slopes of tangent lines.
4. Derivatives of functions.

Interpretations of Derivative at a Point

$$lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Interpretations of Derivative at a Point

$$lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

1. The slope of the graph $y = f(x)$ at $x = a$.
2. The slope of the tangent line to the curve $y = f(x)$ at $x = a$.
3. The rate of change of $f(x)$ with respect to x at $x = a$.
4. The derivative of $f(x)$ at $x = a$.

Derivative as a function

Definition

$$f'(x) = lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

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Alternative Form.

$$f'(x) = lim_{z\to x}\frac{f(z)-f(x)}{z-x}$$

Theorem

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However, using our limit laws, this is equivalent to showing

$$lim_{x \to a}(f(x) - f(a)) = 0.$$

Theorem

If f is differentiable at a, then f is continuous at a.

To prove the theorem we will assume

$$f'(a) = lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

and we will show

$$lim_{x\to a}(f(x)-f(a)) = 0.$$

Formula 1: When c is a constant

$$\frac{d}{dx}(c) = 0$$

Formula 2: When n is a positive integer,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

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fact: $x^n - a^n =$
$(x-a)(x^{n-1} + ax^{n-2} + a^2x^{n-3} + ... + a^{n-2}x + a^{n-1})$

Formula 3:(General Power Rule) When n is any real number,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Formula 4: If c is a constant and f is differentiable, then

$$\frac{d}{dx}(cf(x)) = c\frac{d}{dx}(f(x))$$

Formula 5:(Sum Rule)If g and f are differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$$

Formula 6:(Exponential Functions)

$$\frac{d}{dx}[a^x] = ln(a)a^x$$

Formula 7:(Product Rule) If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}(g(x)) + g(x)\frac{d}{dx}(f(x))$$

Formula 8:(Quotient Rule) If f and g are differentiable, then

$$\frac{d}{dx}[\frac{f(x)}{g(x)}] = \frac{g(x)\frac{d}{dx}(f(x)) - f(x)\frac{d}{dx}(g(x))}{(g(x))^2}$$