# Math 103: Related Rates

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# Outline

1. Review

2. Related Rates

# Important Formulas from Last Time

1. $\frac{d}{dx}(f^{-1}(x)) = \frac{1}{f'(f^{-1}(x))}$
2. $\frac{d}{dx}(ln(x)) = \frac{1}{x}$
3. $\frac{d}{dx}(sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$
4. $\frac{d}{dx}(tan^{-1}(x)) = \frac{1}{1+x^2}$
5. $\frac{d}{dx}(sec^{-1}(x)) = \frac{1}{|x|\sqrt{x^2-1}}$

Related Rates is the most important application of calculus we have seen so far.

**Example** Air is being pumped into a spherical balloon so that its volume increases at a rate of $10\frac{cm^3}{s}$. How fast is the radius of the balloon increasing when the diameter is $4cm$?

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## How To Approach These Problems

1. Draw a picture and name the variables and constants.
2. Write down any additional numerical info.
3. Write down what you are asked to find.
4. Write an equation that relates the quantities.
5. Differentiate with respect to $t$.
6. Finish solving the problem. Remember units.

**Example** A water tank has the shape of an inverted circular cone with base radius 2 meters and a height of 3 meters. If the water is being pumped into the tank at a rate of $3\frac{m^3}{min}$, find the rate at which the water level is rising when the water is 2 meters deep.

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**Example**A ladder 6ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of $.5\frac{ft}{sec}$, how fast is the top of the ladder sliding when it is $1ft$ above the ground?

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**Example** A round oil slick uniformly $0.1cm$ thick is being fed by a leak in an off shore oil rig at a rate of $2\frac{m^3}{sec}$. Sea turtles have bad eyesight and only see the oil as it is nearly on top of them. If sea turtles swim at a rate of $1\frac{m}{sec}$ and begins swimming away from the slick as they see it approaching, how far away from the oil rig does a turtle need to be to avoid being overcome by the slick.

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