Math 103: The Chain Rule and Implicit Differentiation

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Outline

1. Review
2. The Chain Rule
3. Implicit Differentiation

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6. $\frac{d}{dx}(cot(x)) = -(csc(x))^2$

Composite functions

A function $F(x)$ is a **composite** function if it can be written as $F(x) = f(g(x))$ for two functions $f(x)$ and $g(x)$.

Chain Rule

If g is differentiable at x and f is differentiable at $g(x)$, then the composition function $F = f \circ g$ defined by $F(x) = f(g(x))$ is differentiable at x and

$$F'(x) = f'(g(x))g'(x)$$

Most of the functions we have investigated so far can be described by expressing one variable in terms of another explicitly.

1. $y = x^2 + 2$
2. $y = sin(x)$
3. $y = \sqrt{(sin(x))^2 + 1}$

However, some functions are better defined implicitly.

1. $x^2 + y^2 = 1$
2. $y^5 + 3x^2y^2 + 5x^4 = 12$
3. $2(x^2 + y^2)^2 = 25(x^2 - y^2)$
4. $cos(x)sin(y) = 1$

Goal: Find y' without having to solve for y.

Implicit Differentiation

1. Differentiate both sides of the equation with respect to x, treating y as a differentiable function of x.
2. Collect the terms with $\frac{dy}{dx}$ on one side of the equation and solve for $\frac{dy}{dx}$.