



Magnetic
Resonance
Imaging

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The Basic
MR-imaging
Experiment

Cartesian
Sampling

Echoes

Basic Contrast
Mechanisms

Spectral
Imaging

Magnetic Resonance Imaging

II, Fourier Transform, Imaging and Contrast

Charles L. Epstein

Departments of Mathematics
and
Applied Math and Computational Science
University of Pennsylvania

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Outline for Today

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In the last lecture we described the phenomenon of nuclear magnetic resonance and described how, in principle, it could be used to image an object. In medical applications what is actually imaged is the distribution $\rho(\mathbf{x})$ of water protons in an object. Because the nuclear spins are affected by their chemical and physical environment, there are many possible contrast mechanisms, which serve as overlays on the proton density. In today's lecture we describe the details of simple, but realistic imaging experiments. This entails a discussion of the Fourier transform, sampling, resolution, and noise. We introduce pulse-sequence diagrams, used in MR as schematics for imaging protocols. We conclude with a discussion of contrast mechanisms, and a demonstration of Jeremy Magland's Virtual MR-scanner.



The Bloch Equation, reprise

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Recall \mathbf{M} is the bulk magnetization produced by the nuclear spins of protons in water.

$$\frac{d\mathbf{M}}{dt}(\mathbf{x}; t) = \gamma \mathbf{M}(\mathbf{x}; t) \times \mathbf{B}(\mathbf{x}; t) - \frac{1}{T_2} \mathbf{M}^\perp - \frac{1}{T_1} (\mathbf{M}^\parallel - \mathbf{M}_0). \quad (1)$$

$$\mathbf{B}(\mathbf{x}; t) = \mathbf{B}_0(\mathbf{x}) + \mathbf{G}(\mathbf{x}) + \mathbf{B}_1(\mathbf{x}; t).$$

$$\mathbf{M}_0 = \epsilon \rho(\mathbf{x}) \mathbf{B}_0(\mathbf{x}).$$

\mathbf{M}^\parallel is parallel to \mathbf{B}_0 and \mathbf{M}^\perp is orthogonal to \mathbf{B}_0 .



Polarization

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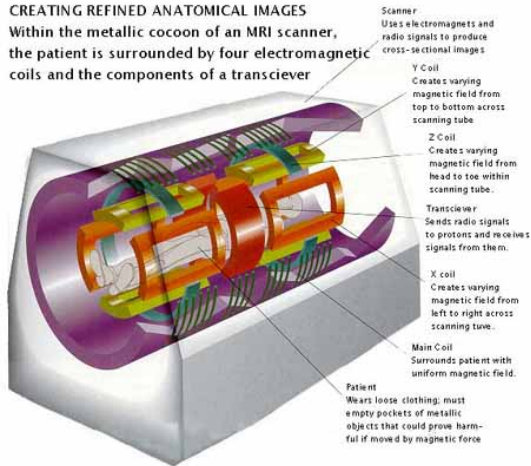
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We suppose that the object to be scanned is placed within the uniform, strong background field, $\mathbf{B}_0 = (0, 0, b_0)$, for a time that is long compared to T_1 . This polarizes the spins, leading to an equilibrium magnetization:

$$\mathbf{M}_0 = \epsilon\rho(\mathbf{x})\mathbf{B}_0(\mathbf{x}).$$

CREATING REFINED ANATOMICAL IMAGES
Within the metallic cocoon of an MRI scanner, the patient is surrounded by four electromagnetic coils and the components of a transceiver





Field Gradients

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As indicated in the previous slide, there are several sets of magnet coils surrounding the object. In addition to the main field coil (which generates \mathbf{B}_0) there are smaller electromagnets that can produce gradient fields, nominally of the form

$\mathbf{G}(\mathbf{x}) = (\star, \star, \langle \mathbf{g}, \mathbf{x} \rangle)$, that is the z -component is a linear function of position. Maxwell's equations require that the field be divergence free and therefore the x - and y -components are not zero, but they can be ignored because \mathbf{B}_0 points along the z -axis and is very large.

The next step is selective excitation: we turn on a gradient field of the form above (with $\mathbf{g}_{ss} = (0, 0, g_z)$ for simplicity). This gives the resonance frequency of the spins a spatial dependence of the form $\omega(\mathbf{x}) = \gamma b_0 + \gamma g_z z = \omega_0 + f$. For the remainder of this discussion we work in the rotating reference frame. That is, we remove the precession about the z -axis at the “reference” Larmor frequency ω_0 .



Selective Excitation

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With the slice-select gradient field $\mathbf{G}_{ss} = (\star, \star, \langle \mathbf{g}_{ss}, \mathbf{x} \rangle)$ turned on, we expose the sample to a spatially uniform RF-field $\mathbf{B}_0(t) = (q(t)e^{i\omega_0 t}, 0)$. The envelope $q(t)$ is chosen so that the magnetization in a range of frequencies $[f_0, f_1]$ is flipped (in the rotating frame) from the equilibrium direction $(0, 0, 1)$ to point along the direction $X_\theta = (\cos \theta, 0, \sin \theta)$; outside a slightly larger band, $[f_0 - \Delta f, f_1 + \Delta f]$, the magnetization remains in the equilibrium state. This is called a selective θ -flip.

Throughout today's talk we assume that we are imaging a 2-dimensional slice at a time. The ideas we present can also be adapted to 3-dimensional imaging schemes.



Rephasing

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At the conclusion of the RF-excitation the slice-select gradient field is turned off. Actually, the magnetization does not quite point in the direction we want,

$$m_x(f; T) + im_y(f; T) = r \left(\frac{f}{2} \right) e^{ifT} (1 + m_z(f; T)),$$

there is a linear phase error. What is actually done is that the slice-select gradient has to be reversed in sign for a time approximately equal to half the time the RF-field is on, then it is turned off. This is called rephasing; it removes the factor e^{ifT} , and leaves the magnetization pointing in the direction X_θ throughout the slice.

This is what is referred to as *phase coherence*, or just coherence.



Signal Equation, I

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Because there is a correspondance between spatial locations and offset frequencies, this means that there is a slice $z_0 < z < z_1$ where the magnetization now points in the direction X_θ , while outside a slightly larger slice the magnetization is still parallel to \mathbf{B}_0 . Only magnetization with a non-zero transverse component will precess in the background field, producing a measurable signal. The measured signal takes the form

$$s(t) \propto \int_D \bar{\rho}(x, y) dx dy, \quad (2)$$

where D is the projection, onto the xy -plane, of the object between $z = z_0$ and $z = z_1$, and

$$\bar{\rho}(x, y) = \int_{z_0}^{z_1} \rho(x, y, z) dz. \quad (3)$$



Signal Equation, II

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We can do much better than the signal described in the previous slide, as this is little more than the total spin density within the selected slice. If, while acquiring the RF-signal, we turn on a gradient field of the form

$$\mathbf{G}_\phi = (\star, \star, \langle g(\cos \phi, \sin \phi, 0), \mathbf{x} \rangle), \quad (4)$$

then the measured signal takes the form

$$s(\phi; t) \propto \int_D e^{i\gamma gt \langle (\cos \phi, \sin \phi), (x, y) \rangle} \bar{\rho}(x, y) dx dy, \quad (5)$$

which is the Fourier transform of $\bar{\rho}$ at the spatial frequency

$$\mathbf{k}(t) = -\gamma gt (\cos \phi, \sin \phi).$$



Signal Equation, III

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By measuring samples of $s(\phi; t)$ and repeating this procedure with a collection of angles $\phi_k = k \Delta\phi$, we can obtain the samples of $\widehat{\rho}$:

$$\{\widehat{\rho}(l \Delta r (\cos k \Delta\phi, \sin k \Delta\phi)) : l = 0, \dots, 2\pi / \Delta\phi, k = 0, \dots, K\}. \quad (6)$$

From this data, we can reconstruction an approximation to $\bar{\rho}$. While such radial sampling is sometimes used in MR, uniform Cartesian sampling is more common. It also makes it easier to discuss questions of sampling and resolution.



x -space and k -space

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The images exist in what we call x - or image-space. The measurements are made in the Fourier domain, which is called k -space. The actual measured signal is modulated by the resonance frequency. Demodulation of this signal is closely parallel to our introduction of the rotating reference frame. The DC-component, after demodulation, of the measured signal corresponds to $(0, 0)$ in k -space.

The resonance frequency has *nothing* whatsoever to do with the spatial resolution available in the reconstructed image. This is determined by the strength of the gradient fields and the time available for sampling, which is limited by the rate of T_2 -decay. This process causes the transverse component of $M(\mathbf{x}; t)$ to shrink. In medical applications T_2 is usually in the range 10-200ms.



Sampling

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We would like to measure samples of $\widehat{\rho}(\mathbf{k})$ of the form

$$\{\widehat{\rho}(m \Delta k_x, n \Delta k_y)\}. \quad (7)$$

The sample spacings, Δk_x , Δk_y are determined by the size of object we wish to reconstruct. This is called the *field-of-view* or *FOV*. If the projection of the object, $\bar{\rho}$ is supported in $D \subset [-L, L] \times [-L, L]$, then the Nyquist-Shannon sampling theorem implies that

$$\Delta k \leq \frac{\pi}{L} \quad (8)$$

is necessary to avoid aliasing. This is because a function supported in this square can be uniquely represented as a Fourier series of the form

$$\bar{\rho}(x, y) = \sum_{m, n=-\infty}^{\infty} \hat{\rho}_{nm} e^{i \Delta k (nx + my)}, \quad (9)$$

only if Δk satisfies this condition.



Aliasing Artifact

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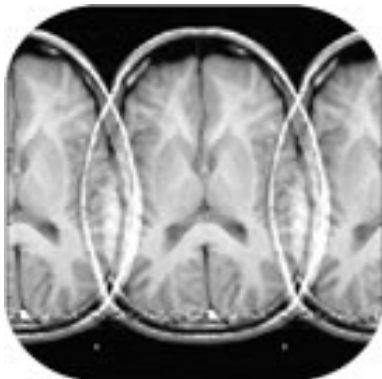
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An image illustrating the result of undersampling in MR-image acquisition.

From: <http://samhess.com/>



Riemann-Lebesgue Lemma

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The data measured in MRI are basically samples of the Fourier transform of $\bar{\rho}$. To a first approximation $\bar{\rho}$ is a piecewise constant function, and this indicates that $\widehat{\bar{\rho}}(\mathbf{k}) = O(\|\mathbf{k}\|^{-1})$. In order to get the best signal-to-noise ratio, we clearly need to collect samples from a neighborhood of $(0, 0)$ in \mathbf{k} -space.

Once the magnetization is flipped into the transverse plane it begins to precess. If there are *no* gradients, imposed or accidental, then this is not a problem, as this precessional motion is coherent across the object and the spins remain in-phase. In the rotating frame, rotating at the resonance frequency, the magnetization is actually stationary (but for T_2 -relaxation).

However, we both impose gradients and have to deal with accidentally occurring gradients. Thus we will need to have methods to *rephase* the spins, or return them to a coherent state.



Spatial Resolution

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We need to decide which region of \mathbf{k} -space we wish to sample. That is we need to choose $k_{x \max}$ and $k_{y \max}$. For simplicity we suppose they are equal. This choice effectively determines the maximum available spatial resolution in the reconstructed image, for

$$\bar{\rho}(x, y) \approx \sum_{m, n = -k_{\max} / \Delta k}^{k_{\max} / \Delta k} \hat{\rho}_{nm} e^{i \Delta k (nx + my)}, \quad (10)$$

contains *no* information about features with spatial frequencies greater than k_{\max} . Though we have omitted it from the model, recall that the signal is decaying like e^{-t/T_2} . As $\|\mathbf{k}(t)\| = t \gamma g$, for a given gradient strength, g , the T_2 -decay rate determines the maximum frequency that can be usefully sampled. In the radial sampling scheme T_2 -decay looks like a form of low-pass filtering, which progressively attenuates the higher frequencies. But recall that T_2 depends on x .



Phase Encoding

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To measure uniformly spaced Cartesian samples, we need to make somewhat different usage of the gradient fields. This entails two steps: these are called the *phase encoding* step and the *frequency encoding*, or *readout* step. For this discussion, we use the k_y -direction as the phase encode direction; this means we first turn on a field of the form $\mathbf{G}_{\text{ph}} = (\star, \star, \langle g(0, 1, 0), \mathbf{x} \rangle)$, for a period of time τ_{ph} so that

$$m \Delta k = \tau_{\text{ph}} \gamma m \Delta g.$$

Different amounts phase in this direction are obtained by stepping the strength of the gradient field; here m takes values in the set $\{-M, \dots, M\}$, so that $M \Delta k = k_{\text{max}}$.

This means that τ_{ph} is fixed, which is important because of the T_2 -relaxation.



Frequency Encoding

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The final step is the frequency encoding or readout step. For this purpose, we first turn on a gradient of the form

$$\mathbf{G}_{\text{fr}} = (\star, \star, \langle -g(1, 0, 0), \mathbf{x} \rangle),$$

for a certain period of time. This moves us to $(-k_{\text{max}}, m \Delta k)$ in \mathbf{k} -space. We then reverse the sign of this field and begin to measure the signal. The measured signal has the form

$$s(t) \propto \int_D e^{im \Delta ky} e^{i(tg\gamma - M \Delta)x} \bar{\rho}(x, y) dx dy. \quad (11)$$

Sampling this signal we obtain approximations to

$$\{\widehat{\bar{\rho}}(n \Delta k, m \Delta k) : -M \leq n \leq M\}. \quad (12)$$

This method of sampling is called a *Gradient Recalled Echo* or *GRE*.



The GRE Pulse Sequence Diagram

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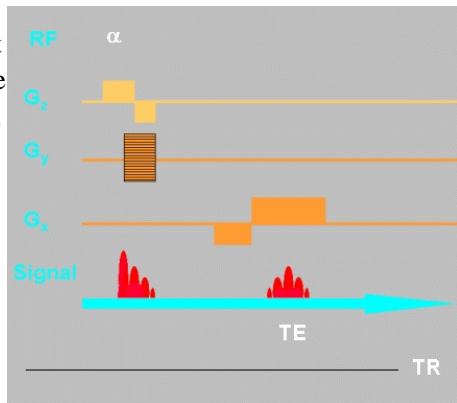
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The experiment described above has many steps, and is difficult to describe concisely. MR-scientists have invented a very simple schematic description of such experiments.



The GRE pulse sequence diagram.

Dr Gary P. Liney © 2003-2005

From: http://www.hull.ac.uk/mri/lectures/gpl_page.html



Receiver Bandwidth and Noise

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The sampling spacing Δk in \mathbf{k} -space, and the strength of the readout gradient, g , determine the *time*-spacing Δt between the measurements. The measured signal is actually an average

$$s_j \approx \frac{1}{\Delta t} \int_{t_j}^{t_j + \Delta t} s(\sigma) d\sigma. \quad (13)$$

This low-pass filter is very important: the MR-signal is very small and noisy.



Noise in MR

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The noise arises because the objects being imaged are electrically conductive. The body in the scanner is the principal source of measurement noise, with the noise variance proportional to the temperature. The noise is essentially white, and so it is quite important that the receiver bandwidth is finite $B \simeq 1/\Delta t$.

In MR, the SNR is proportional to the resonance frequency of the background field (at least in the range of field strengths currently employed)

$$\text{SNR} \propto \omega_0 \propto \|\mathbf{B}_0\|.$$

Measurements are often repeated and averaged to reduce the noise. Averaging N independent measurements reduces the noise by a factor of $1/\sqrt{N}$.



The Echo Time

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In the GRE pulse diagram we saw a time marked as T_E , which occurs when the accumulated phase in the x -direction passes through zero. This is called the *echo time*, it is usually when the signal reaches its maximum amplitude. The experiment described above is called a *gradient recalled echo*, since all the manipulation of the phase is accomplished using gradient fields.

From the earliest days of NMR spectroscopy it was recognized that there were other sources of phase dispersion, or dephasing, which could not be corrected using gradient fields alone. Indeed small inhomogeneities in the background field are the main culprit and lead to a much faster signal decay than that predicted by the theory underlying the “spin-spin” mechanism responsible for the T_2 -decay. In MR this is called T_2^* -decay. Erwin Hahn discovered a very clever trick to recover this apparently lost signal.



Inhomogeneous Dephasing, I

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For this discussion we ignore relaxation effects. The calculations below are in the rotating frame with $\omega_0 = \gamma \|\mathbf{B}_0\|$.

We begin by assuming that the sample is polarized in a field of the form $\mathbf{B}_0 + \Delta\mathbf{B}$, where $\|\Delta\mathbf{B}\| \ll \|\mathbf{B}_0\|$. Because we are measuring differences in phase accumulation, even inhomogeneities with $\|\Delta\mathbf{B}\|/\|\mathbf{B}_0\| \approx 10^{-6}$ can produce significant artifacts and reductions in signal intensity. Without any applied gradients we apply an RF-pulse that flips the spins by 90° , so that at the end of the RF-excitation the magnetization satisfies $\mathbf{M}(\mathbf{x}) = m(\mathbf{x})(1, 0, 0)$.

This magnetization will now begin to precess in the local field so that after time t the field takes the form

$\mathbf{M}(\mathbf{x}; t) = m(\mathbf{x})(e^{it\Delta\phi(\mathbf{x})}, 0)$, where

$$\Delta\phi(\mathbf{x}) = \gamma \Delta\mathbf{B}(\mathbf{x}). \quad (14)$$

If $\Delta\mathbf{B} \equiv 0$, then all the spins would remain in phase.



Inhomogeneous Dephasing, II

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Recall that $\gamma \approx 2.675 \times 10^8 \text{ rad/sT}$. If $\mathbf{B}_0 = 3\text{T}$, then a 1 p.p.m. (very small!) inhomogeneity in \mathbf{B} leads to phase error accumulation at a rate of about 1000 rad/s. MR-measurement times are usually measured in milliseconds. So in 10ms we would accumulate 10 rads of phase error. The function $\Delta \mathbf{B}(\mathbf{x})$ usually varies on a very small length scale and this can lead to a great deal of phase dispersion and signal loss in a very short period of time. There are many sources of field inhomogeneity of this magnitude, some produced by the body itself.



T_2^* Decay

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These inhomogeneous dephasing effects are usually modeled as changing the spin-spin relaxation time from T_2 to a smaller T_2^* , sometimes *much* smaller. Thus, without some corrective measures, we observe that

$$\|(M_x + iM_y)(t)\| = O(e^{-\frac{t}{T_2^*}}),$$

rather than

$$\|(M_x + iM_y)(t)\| = O(e^{-\frac{t}{T_2}}).$$

This is often called T_2^* -decay.

This actually defines a new image contrast, that can sometimes be quite distinct from T_2 -decay alone.

In order not to lose signal strength to *inhomogeneous dephasing*, we require a non-gradient based method to restore the phase coherence.



The Hahn Echo, I

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Mathematically, it is very easy to see what is needed. Suppose we can find a RF-pulse which implements the inversion map

$$\mathcal{I} : (x + iy, z) \mapsto (x - iy, -z). \quad (15)$$

This is an element of $SO(3)$, so there is no reason why we should not be able to do this. If we apply this map at time T to the magnetization it would have the effect

$$\mathcal{I} : m(\mathbf{x})(e^{iT\Delta\phi(\mathbf{x})}, 0) \mapsto m(\mathbf{x})(e^{-iT\Delta\phi(\mathbf{x})}, 0). \quad (16)$$

Assuming that the spins are stationary, the phase will continue to accumulate in the *same* inhomogeneous field so that, for $t > T$,

$$\mathbf{M}(\mathbf{x}; t) = m(\mathbf{x})(e^{i(t-2T)\Delta\phi(\mathbf{x})}, 0).$$

When $t = 2T$ the magnetization is completely rephased, and once again coherent: $\mathbf{M}(\mathbf{x}; 2T) = m(\mathbf{x})(1, 0)$. The time $2T$ is called the *echo time*, T_E .



The Hahn Echo, II

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This method for restoring phase coherence is called the Hahn Echo, and it provides a powerful technique in NMR for improving the signal-to-noise ratio and generally reducing artifacts due to inhomogeneity, chemical shift, etc. But how does one do this in practice?

In the situation described above we only need to invert the spins at a single frequency, in the presence of the strong RF-pulse the small variations on Larmor frequency caused by $\Delta \mathbf{B}$ are unimportant. In this case we just use the pulse

$$\mathbf{B}_1 = q(1, 0, 0) \quad (17)$$

for a time τ selected so that $\gamma q \tau = \pi$.



Selective Inversion

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But what if we want to invert spins in a frequency selective manner? This is somewhat trickier, for we have the non-linear Parseval inequality for the pulse $(q(t), 0)$:

$$\int_0^T |q(t)|^2 dt \geq \frac{1}{\pi} \int_{-\infty}^{\infty} \log(1 + |r(\xi)|^2) d\xi. \quad (18)$$

At any frequency ξ where the magnetization is inverted $|r(\xi)| = \infty$, so we can only do this approximately over a frequency band of positive width. Using one of several methods, one can design such selective inversion pulses.



An Inversion Pulse

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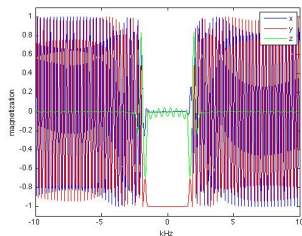
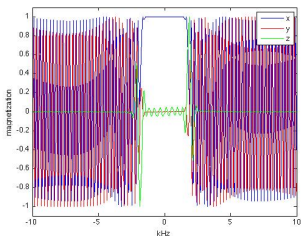
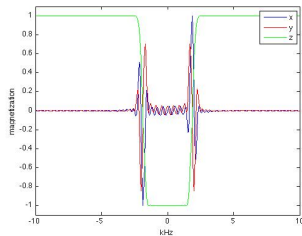
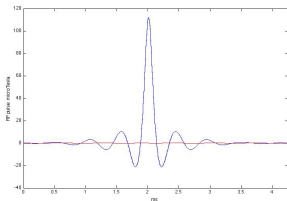
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A frequency selective inversion pulse and the response of the z-, x- and y-components.



The Spin Echo Experiment

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For the spin-echo experiment we insert an inversion pulse (frequency selective or not) after the phase encoding. We then turn on the frequency encoding gradient as before, with a negative lobe to move us to $(-k_{\max}, m \Delta k)$ in k -space; after reversing the polarity of the read-out gradient we acquire the signal. The result is similar to the GRE, though the spin-echo refocuses all of the inhomogeneity and chemical shift induced dephasing.



Pulse Diagram for the Spin-Echo Experiment

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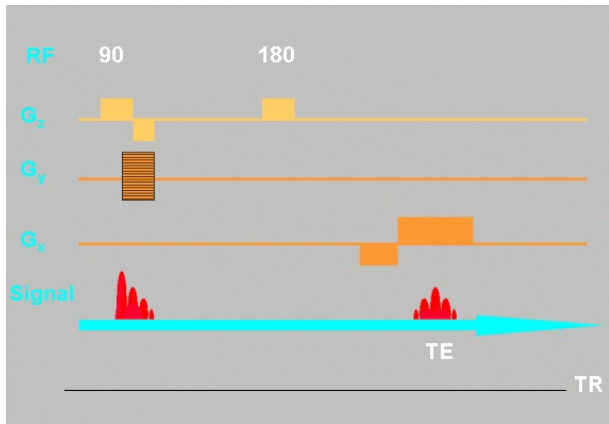
The Basic MR-imaging Experiment

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The Spin Echo pulse sequence diagram.

Dr Gary P. Liney © 2003-2005

From: http://www.hull.ac.uk/mri/lectures/gpl_page.html



Proton Density, T_1 and T_2 Weighted Images

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As remarked yesterday, the utility of any medical imaging modality lies in the contrast one can obtain in the image. Because the phase of a spin is so exquisitely sensitive to the local electronic environment at the nucleus, MRI provides an unparalleled array of potential contrast mechanisms. Too much sensitivity would make the method unstable, but as in so many other things for MR, this is also just right.

In today's lecture we discuss the three simplest contrast mechanisms, or *weightings*, and describe how to obtain images displaying these contrasts. Ignoring all relaxation and chemical shift phenomena, we would expect to be able to directly image the proton density $\rho(\mathbf{x})$. This is in fact the case. By making small modifications in the imaging protocol, we can also get images that overlay the proton density with the spatial variation of either T_1 , or T_2 separately or combined in various ways. This flexibility has proved to be enormously important in clinical applications of MRI.



Basic Assumptions

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Recall that, in the absence of RF-fields, the Bloch equations predict that the longitudinal component returns to the equilibrium according to

$$M_z(t) = M_{0z} + (M_z(0) - M_{0z})e^{-\frac{t}{T_1}}.$$

The transverse components decay like

$$M_{xy}(t) = M_{xy}(0)e^{-\frac{t}{T_2}}. \quad (19)$$

In liquids the relaxation mechanisms are different for the two components, and $T_2 < T_1$. Both are in the 10s of milliseconds range.

In the discussion that follows we assume that signal acquisition step is sufficiently fast compared to T_2 , that relaxation can be ignored during actual measurement process.



Experimental Parameters, I

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There are many possible parameters we can choose in an MR-imaging experiment, and these choices affect the sort of contrast exhibited in the image.

To simplify the discussion we assume that the magnetization in a slice of fixed thickness is flipped by an angle $\theta \leq 90^\circ$, after which we immediately rephase and use the phase encoding gradient to fix a line in k -space that will be measured in the acquisition step. We assume that the sample spacing and resolution are also fixed.



Experimental Parameters, II

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- This brings us to the first important choice: how long do we wait before making a measurement. This is called the echo time T_E .
- After we measure a line in k -space, we then need to decide how long to wait before repeating the process of selective excitation/phase encoding. This is called the repeat time T_R .
- The choice of T_E and T_R largely determine the sort of image contrast we obtain.
- A short T_E does not allow for much T_2 -relaxation, whereas a long T_1 allows the magnetization to return to equilibrium.



Proton Density Weighted Imaging

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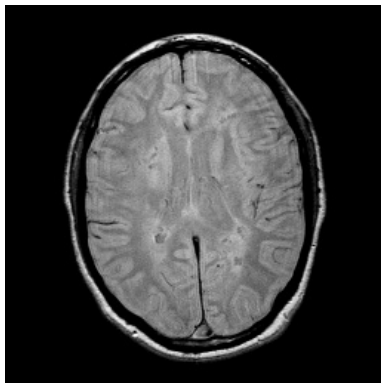
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A short echo time
 $T_E \ll T_2$ means
that little
 T_2 -relaxation will
have occurred, and
a long repeat time
 $T_R \gg T_1$ means
that the
longitudinal
component will
have returned to its
equilibrium value.
This produces a
proton density
image.



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From:

http://www.hull.ac.uk/mri/lectures/gpl_page.html



T_1 -weighted Imaging

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A short echo time $T_E \ll T_2$ means that little T_2 -relaxation will have occurred, and a repeat time T_R comparable to T_1 means that the size of the longitudinal component will depend on $T_1(x)$. This produces a T_1 -weighted image. Due to the short T_E , the residual transverse magnetization must be destroyed before we repeat the acquisition.



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T_2 -weighted Imaging

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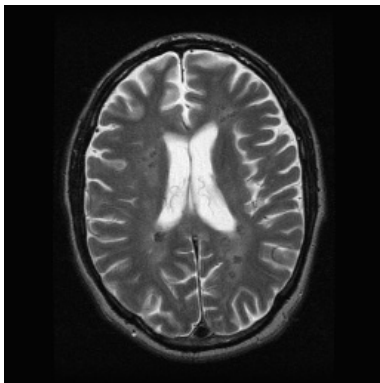
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A long echo time, $T_E > T_2$, means that considerable T_2 -relaxation will have occurred; combined with a long repeat time $T_R \gg T_1$, this produces a T_2 -weighted image.



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Diagnosis based on appearance with T_1 -weighting

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Dark on T1-weighted image:

- * increased water, as in edema, tumor, infarction, inflammation, infection, hemorrhage (hyperacute or chronic)
- * low proton density, calcification
- * flow void

Bright on T1-weighted image:

- * fat
- * subacute hemorrhage
- * melanin
- * protein-rich fluid
- * slowly flowing blood
- * paramagnetic substances: gadolinium, manganese, copper
- * calcification (rarely)
- * laminar necrosis of cerebral infarction

From: Neuroimaging Primer, Keith A. Johnson, M.D., Harvard Medical School at <http://www.med.harvard.edu/AANLIB/basicsMR.html>



Diagnosis based on appearance with T_2 -weighting

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Bright on T2-weighted image:

- * increased water, as in edema, tumor, infarction, inflammation, infection, subdural collection
- * methemoglobin (extracellular) in subacute hemorrhage

Dark on T2-weighted image:

- * low proton density, calcification, fibrous tissue
- * paramagnetic substances: deoxyhemoglobin, methemoglobin (intracellular), iron, ferritin, hemosiderin, melanin
- * protein-rich fluid
- * flow void

From: Neuroimaging Primer, Keith A. Johnson, M.D., Harvard Medical School at <http://www.med.harvard.edu/AANLIB/basicsMR.html>



The Chemical Shift

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In our introduction to NMR we described how different chemical environments lead to slightly different magnetic field strengths where hydrogen protons are located. Since the resonance frequency is proportional to this field strength, the spins in protons in different chemical environments will precess at slightly different frequencies. This shift in the resonance is called a *chemical shift*. There are also different relaxation rates. If a sample of a chemical compound, with N types of spins, is placed in an strong homogeneous field, and the magnetization produced by the nuclear spins is excited, we will measure a signal, called a *free induction decay* (FID) of the form:

$$s(t) = e^{-\omega_0 t} \sum_{j=1}^N \rho_j e^{i\Delta\omega_j t} e^{-\frac{t}{T_{2j}}} . \quad (20)$$



An FID and its Fourier Transform

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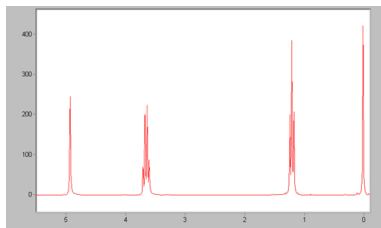
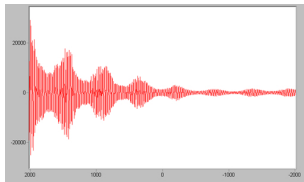
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The Fourier transform of the FID shows the chemical shifts present in the sample. The widths of the peaks are proportional to T_{2j}^{-1} . The relative areas under these peaks reflects the relative sizes of the populations of each type of spin. This information can often be used to identify chemical compounds.



Spectral Imaging, I

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- The techniques used to acquire spatially localized information can be combined with the measurement of the FID to obtain images depicting the distribution of chemical compounds. This is called *magnetic resonance spectroscopy* (MRS).
- A simple MRS experiment could consist of the following steps:
 - 1 Selectively excite and rephase the spins in a slice (with a sufficiently wide bandwidth to capture all relevant chemical shifts).
 - 2 Use gradients to phase encode in both the k_x and k_y directions.
 - 3 With no gradient field measure the FID.
 - 4 Repeat with different phase encodes.



Spectral Imaging, II

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The data measured by the procedure described in the previous slide is of the form $\{\widehat{\rho}_{nm}(t) : -N \leq n, m \leq N\}$. If we invert the Fourier series, then

$$\rho_{jk}(t) = \sum_{n,m} \widehat{\rho}_{n,m}(t) e^{i\Delta k(nx_j + my_k)} \quad (21)$$

is the FID from the pixel at spatial location (x_j, y_k) . Computing its Fourier transform give the NMR spectrum of the material located in this pixel.

Note: The design of MR-experiments is modular.



A Clinical Example of MRS

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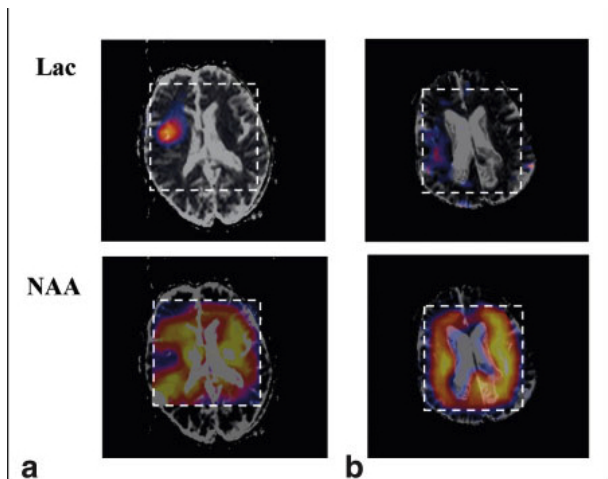
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MRS images showing levels of lactate (Lac) and N-acetylaspartate (NAA) in the brains of stroke patients.



A Clinical Example of MRS, bis

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From: *Multiple Spin-Echo Spectroscopic Imaging for Rapid Quantitative Assessment of N-Acetylaspartate and Lactate in Acute Stroke*, by Astrid Stengel, Tobias Neumann-Haefelin, Oliver C. Singer, Claudia Neumann-Haefelin, Friedhelm E. Zanella, Heinrich Lanfermann, and Ulrich Pilatus, in *Magnetic Resonance in Medicine* 52:228-238 (2004)



Conclusion

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We have shown a small and elementary selection of the known contrast mechanisms used in MR-imaging. In the next lecture we show how to use MR to measure the diffusion constant and tensor.



Some good MR-websites

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- A comprehensive online MR-textbook
<http://www.cis.rit.edu/htbooks/mri/inside.htm>
- An online MR-textbook:
http://www.hull.ac.uk/mri/lectures/gpl_page.html
- Another online MR-textbook:
<http://www.ebme.co.uk/arts/mri/index.htm>
- The whole brain atlas:
<http://www.med.harvard.edu/AANLIB/home.html>
- A good site for fMRI:
<http://www.fmrib.ox.ac.uk/education/fmri>
- Wikipedia also has a good article:
http://en.wikipedia.org/wiki/Magnetic_resonance_imaging