Math 584, Problem set 6 due April 25, 2017
Dr. Epstein

Reading: Read Chapters 9.1–9.1.6, 9.2, 10.1–10.3 and 10.5, 11.1–11.4. Prepare your class presentation. The written version should be handed in no later than April 29.

(1) From the text do problem: 8.1.4.

(2) From the text do problem: 8.2.8, 8.2.10, 8.2.13.

(3) From the text do problem: 8.3.3, 8.3.4

(4) Suppose that f is an L-bandlimited function and g is an M-bandlimited function, show that pointwise product $h(x) = f(x)g(x)$ is an $L + M$-bandlimited function.

(5) Let $f(x) = \frac{1-\cos(2x)}{\pi x^2}$.

(a) Show that this function is bandlimited and determine the Nyquist sampling rate for it.

(b) Suppose that we sample at half the Nyquist rate and apply the Shannon-Whittaker formula to obtain a function $F(x)$ which interpolates the samples and whose Fourier transform is supported in a band of half the length. What is $\widehat{F}(\xi)$?

(c) Graph $f(x)$ and $F(x)$ on the same plot.

(6) Do exercises 9.2.5, 9.2.7 from the book.

(7) Do exercises 10.2.2, 10.2.3 from the book.

(8) Recall Simpson's rule: if $f_j = f(a+jd)$, where $d = (b-a)/N$, (N an even number), then

$$\int_a^b f(x) \approx \frac{d}{3}[f_0 + 4f_1 + 2f_2 + 4f_3 + \ldots + 2f_{N-2} + 4f_{N-1} + f_N].$$

Explain how to modify the definitions of the zero padded sample sequences, $< f_0, \ldots, f_N, 0, \ldots, 0 >$, $< h_0, \ldots, h_N, 0, \ldots, 0 >$, to use the $(2N-1)$-point DFT to give a Simpson's rule approximation for the samples of the convolution:

$$\int_0^1 h\left(\frac{k}{N} - y\right) f(y)dy.$$