Math 241 Final Exam
Spring 2003
University of Pennsylvania

May 1, 2003

No books, tables or calculators allowed. One 8 ½''x11'' sheet of notes is permitted. You must show work to get credit. All questions are worth 10 points with no partial credit. **Circle the entire answer you deem correct**. Write your information below; write your name on every page.

Name: ____

Signature: _____________________________________

Penn ID#: _____________________________________

Please do not write below this line

SCORE: ____________

Name:________________________________

1) Which of the following are two solutions to the equation $(z-2)^4=-1$

a) $e^{\pi i/4}$ and $\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$

b) $-2+e^{\pi i}$ and $\left(-2+\frac{\sqrt{2}}{2}\right)+\frac{\sqrt{2}}{2}i$

c) $-2+e^{\pi i/4}$ and $\left(-2+\frac{\sqrt{2}}{2}\right)-\frac{\sqrt{2}}{2}i$

d) $2+e^{\pi i/2}$ and $2-i$

e) $2+e^{3\pi i/4}$ and $\left(2-\frac{\sqrt{2}}{2}\right)-\frac{\sqrt{2}}{2}i$

f) $2+e^{5\pi i/4}$ and $\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$

2) Evaluate $\oint_C \bar{z}\,dz$ where C is the circle of radius 3 centered at 0.

a) $2\pi i$

b) $6\pi i$

c) $12\pi i$

d) $18\pi i$

e) 0

f) Does not exist

Name:_______________________________

3) Suppose that $\sum_{n=0}^{\infty} a_n (x-1)^n$ is the Taylor series centered at 1 for the real valued function $f(x) = \dfrac{1}{x^2+2x+2}$. Which of the following is true about the radius of convergence R of this series?

a) $R=0$

b) $R=1$

c) $R=\sqrt{2}$

d) $R=2$

e) $R=\sqrt{5}$

f) $R=3$

g) R is infinite

4) Evaluate $\int_\gamma \dfrac{1+z}{1-\cos z}dz$ where γ is the circle centered at $\dfrac{1}{2}$ with radius 1.

a) $6\pi i$

b) $4\pi i$

c) $2\pi i$

d) 0

e) $-2\pi i$

f) None of these.

Name:________________________________

5) Evaluate the real integral $\int_0^{2\pi} \frac{d\theta}{5-4\cos\theta}$.

a) $\frac{2\pi}{3}$

b) $-\frac{2\pi}{3}$

c) $2\pi i$

d) $-\frac{2\pi i}{3}$

e) $\frac{4\pi}{3}$

f) None of these.

6) Suppose that $f(x)$ is periodic with period 4 and that for $-2<x<2$ the function is defined to be $f(x)=\begin{cases} -x & \text{for } -2<x<0 \\ x & \text{for } 0<x<2 \end{cases}$. Which of the following is the coefficient of $\cos\frac{3\pi x}{2}$ in the Fourier Series of $f(x)$:

a) $-\frac{8}{\pi^2}$

b) $\frac{8}{\pi^2}$

c) $\frac{8}{9\pi^2}$

d) $-\frac{8}{9\pi^2}$

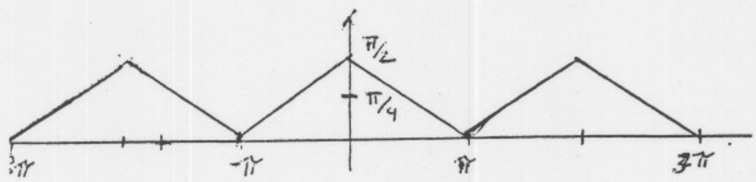
e) $\frac{8}{3\pi^2}$

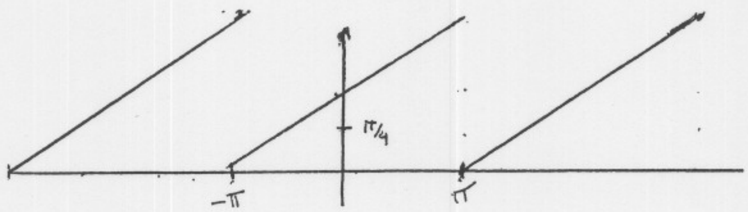
f) None of these.

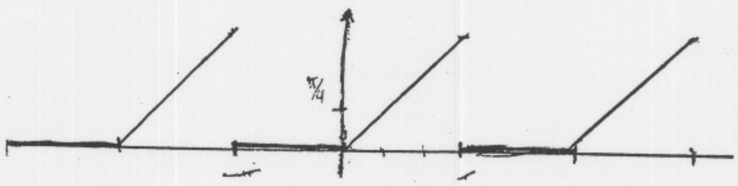
Name:_______________________________

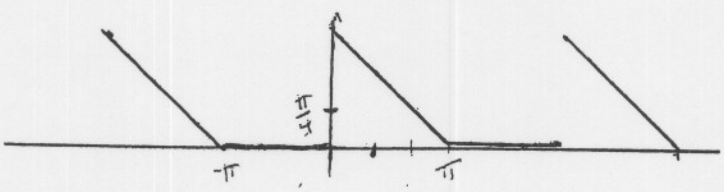
7) Suppose that a periodic function $g(x)$ with period 2π is represented by the Fourier Series $\frac{\pi}{4}-\frac{2}{\pi}\left(\cos x+\frac{1}{9}\cos 3x+\frac{1}{25}\cos 5x+\ldots\right)+\left(\sin x-\frac{1}{2}\sin 2x+\frac{1}{2}\sin 3x-\ldots\right)$.

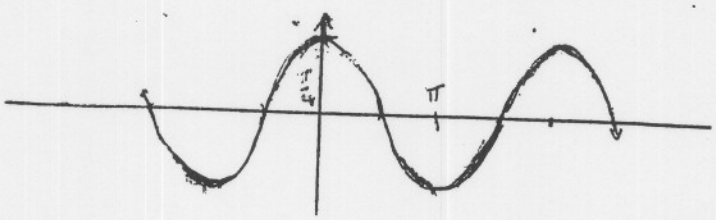
Which of the following is the graph of $g(x)$?

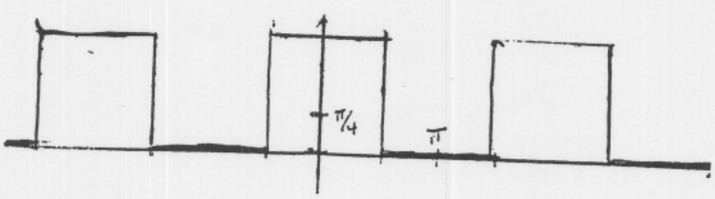
a) 

b) 

c) 

d) 

e) 

f) 

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8) Heat flow on a bar of length π is given by the partial differential equation $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$ for $0 < x < \pi$ and $t > 0$ with boundary conditions $u(0,t) = u(\pi,t) = 0$ and initial conditions $u(x,0) = \begin{cases} x & \text{for } 0 < x < \pi/2 \\ \pi - x & \text{for } \pi/2 < x < \pi \end{cases}$. Find the coefficient of $e^{-50t}\sin 5x$ in the solution.

a) $\frac{4}{25\pi}$

b) $\frac{1}{25}$

c) $\frac{1}{25\pi}$

d) $-\frac{1}{9}$

e) $\frac{4}{9\pi}$

f) None of these

9) Which one of the following is a solution of the partial differential equation $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$ where $u(0,t) = u(\pi,t) = 0$ for all t?

a) $e^{2x}\cos t$

b) $e^{x/2}e^{t}$

c) $(\sin 3x)(\sin 6t)$

d) $(\sin 7x)e^{14t}$

e) $\left(\sin\frac{3x}{2}\right)(\cos 3t)$

f) $\left(\cos\frac{5x}{2}\right)(\sin 5t)$

Name:________________________________

10) Suppose that $u(x,y)$ satisfies Laplace's equation $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ on the rectangle $0 \le x \le 1, 0 \le y \le 2$ satisfying the boundary conditions $u(x,0) = u(x,2) = u(0,y) = 0$ and $u(1,y) = \sin 3\pi y$. Find $u\left(\frac{1}{2},\frac{1}{2}\right)$:

a) $\dfrac{\left(e^{3\pi/2} - e^{-3\pi/2}\right)}{\left(e^{3\pi} - e^{-3\pi}\right)}$

b) $e^{3\pi/2} - e^{-3\pi/2}$

c) $e^{3\pi} - e^{-3\pi}$

d) 1

e) 0

f) None of these.

11) The Laurent Series for the function $\frac{z}{z^3-1}$ valid on the annulus $\frac{1}{2} < |z-1| < 1$ is ….

a) $\dfrac{1}{3(z-1)} + 1 + \frac{1}{9}(z-1) + \frac{1}{9}(z-1)^2 + \ldots$

b) $\dfrac{-1}{3(z-1)} - 1 - \frac{1}{9}(z-1) + \frac{1}{9}(z-1)^2 + \ldots$

c) $\dfrac{1}{3(z-1)} + \frac{1}{9}(z-1) + \frac{1}{9}(z-1)^2 + \ldots$

d) $\dfrac{1}{3(z-1)} - \frac{1}{9}(z-1) + \frac{1}{9}(z-1)^2 + \ldots$

e) $\dfrac{1}{3(z-1)} - \frac{1}{9}(z-1) - \frac{1}{9}(z-1)^2 + \ldots$

f) None of these.

Name:________________________________

12) For the Fourier Series for the function $f(x) = \sin^4 x$, which of the following is true?

a) $a_1 = b_1 = 0, a_2 = -\frac{1}{2}, a_3 = 1$

b) all coefficients a_k are zero.

c) $a_2 = -\frac{1}{2}, a_4 = \frac{1}{8}$, and all coefficients b_k are zero.

d) $a_1 = \frac{1}{2}, a_2 = -\frac{1}{2}$, and all coefficients b_k are zero.

e) $a_3 = a_4 = \frac{1}{8}$, and all other coefficients are zero.

f) None of the above.

13) For the region which is the interior of the circle or radius 3 with center at 0, a complex function is defined by $f(z) = \frac{1}{2\pi i} \int_\gamma \frac{w \cdot \overline{w}}{w - z} dw$, where γ is the boundary of the region. Then at $z = \frac{1}{2}$...

a) $f\left(\frac{1}{2}\right) = f'\left(\frac{1}{2}\right) = 0$

b) $f'\left(\frac{1}{2}\right) = f''\left(\frac{1}{2}\right) = 0$

c) $f\left(\frac{1}{2}\right) = f''\left(\frac{1}{2}\right) = 0$

d) $f'\left(\frac{1}{2}\right) = 0$, but $f\left(\frac{1}{2}\right)$ and $f''\left(\frac{1}{2}\right)$ are not 0.

e) $f''\left(\frac{1}{2}\right) = 0$, but $f\left(\frac{1}{2}\right)$ and $f'\left(\frac{1}{2}\right)$ are not 0.

f) None of these

Name:______________________________

14) The vibrations of a certain string of length π satisfy the partial differential equation $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, have the geometric boundary conditions $u(0,t) = u(\pi,t) = 0$ for all t, and satisfy the initial conditions $u(x,0) = 0$ for all x, and $\frac{\partial u}{\partial t}(x,0) = \sin x$. Then $u\left(\frac{\pi}{2}, \frac{\pi}{3}\right)$ is:

a) $-\frac{\sqrt{3}}{2}$

b) $\frac{\sqrt{3}}{2}$

c) $\frac{1}{2}$

d) 0

e) $\sqrt{3}$

f) $-\sqrt{3}$

15) For the function defined by $f(x) = ix$, for $0 < x < 2\pi$, extended to be 2π -periodic, the complex Fourier Series is:

a) $i\pi + \sum_{k=1}^{\infty} \frac{1}{k}\left(e^{ikx} - e^{-ikx}\right)$

b) $i\pi + \sum_{k=1}^{\infty} \frac{1}{k} e^{ikx}$

c) $i\pi - \sum_{k=1}^{\infty} \frac{1}{k^2}\left(e^{ikx} - e^{-ikx}\right)$

d) $i\pi - \sum_{k=1}^{\infty} \frac{1}{k^2}\left(e^{-ikx}\right)$

e) $i\pi - \sum_{k=1}^{\infty} \frac{1}{k}\left(e^{ikx} - e^{-ikx}\right)$

f) None of these.

Name:______________________________

16) The value of the integral $\int_0^\infty \frac{x^2 dx}{1+x^4}$ is

a) $\pi\sqrt{2}$

b) $\frac{\pi\sqrt{2}}{2}$

c) $\frac{\pi\sqrt{2}}{4}$

d) $\frac{\pi}{2}$

e) $2\pi\sqrt{2}$

f) π

17) The sum of the power series $\sum_{n=0}^{\infty}(2n+1)z^{2n}$, valid for $|z|<1$, is the function…

a) $\frac{z^2}{1-z^2}$

b) $\frac{z^2}{1+z^2}$

c) $\frac{1+z^2}{1-z^2}$

d) $\frac{1+z^2}{\left(1-z^2\right)^2}$

e) $\frac{1-z^2}{\left(1+z^2\right)^2}$

f) None of these.

18) The value of the integral $\frac{1}{2\pi i}\oint_C \frac{(\cos z)dz}{(z-(\pi/4))^2}$, where C is the unit circle, $|z|=1$, is:

a) $-\frac{\sqrt{2}}{4}$

b) $\frac{\sqrt{2}}{4}$

c) $-\frac{\sqrt{2}}{2}$

d) $\frac{\sqrt{2}}{2}$

e) $-\frac{1}{2}$

f) $\frac{1}{2}$

19) The residue of the function $\frac{1}{(z-1)(z^3-1)}$ at $z=1$ is:

a) $\frac{2}{3}$

b) $-\frac{2}{3}$

c) $\frac{1}{3}$

d) $-\frac{1}{3}$

e) 0

f) None of these.

Name:________________________________

20) Two values of $\left(\sqrt{2}\right)^{i}$ are:

a) $\left(\cos(\log 2)+i\sin(\log 2)\right)$ and $e^{2\pi}\left(\cos(\log 2)+i\sin(\log 2)\right)$

b) $\left(\cos\left(\frac{1}{2}\log 2\right)+i\sin\left(\frac{1}{2}\log 2\right)\right)$ and $e^{-6\pi}\left(\cos\left(\frac{1}{2}\log 2\right)+i\sin\left(\frac{1}{2}\log 2\right)\right)$

c) $\left(\cos\left(\frac{1}{2}\log 2\right)+i\sin\left(\frac{1}{2}\log 2\right)\right)$ and $e^{3\pi}\left(\cos\left(\frac{1}{2}\log 2\right)+i\sin\left(\frac{1}{2}\log 2\right)\right)$

d) $\left(\cos(\log 2)+i\sin(\log 2)\right)$ and $e^{\pi}\left(\cos(\log 2)+i\sin(\log 2)\right)$

e) $e^{\pi}\left(\cos\left(\frac{1}{2}\log 2\right)+i\sin\left(\frac{1}{2}\log 2\right)\right)$ and $e^{2\pi}\left(\cos\left(\frac{1}{2}\log 2\right)+i\sin\left(\frac{1}{2}\log 2\right)\right)$

f) None of these.