Math 241 Final Exam
Spring 2009

Multiple Choice. Circle the correct answer. Partial credit will be given so be sure to show work. A correct answer with little or no supporting work will receive little or no credit.

1. Evaluate $\int_0^{2\pi} \frac{d\theta}{(5+3\cos\theta)^2}$.

(A) 0 (C) $\frac{\pi}{4}$ (E) $\frac{7\pi}{16}$ (G) $\frac{7\pi}{64}$

(B) $\frac{5\pi}{32}$ (D) $\frac{\pi}{8}$ (F) $\frac{\pi}{3}$ (H) none of these

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2. Find the principal value of $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+2x+2)(x^2+1)}dx$

(A) 0 (C) $\frac{\pi}{4}$ (E) $\frac{3\pi}{5}$ (G) $\frac{2\pi}{3}$

(B) $\frac{2\pi}{5}$ (D) $\frac{\pi}{6}$ (F) $\frac{\pi}{3}$ (H) none of these

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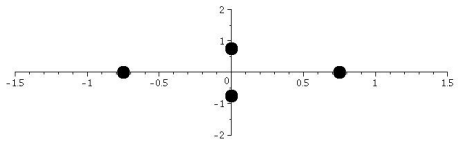
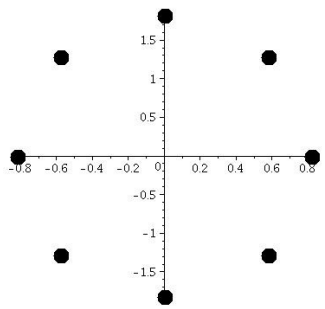
3. If a and b are real-valued, and n is an integer, which of the following plots may NOT indicate the solution set to the following equation? (Please verbally justify your answer.

$$z^n = a+bi$$

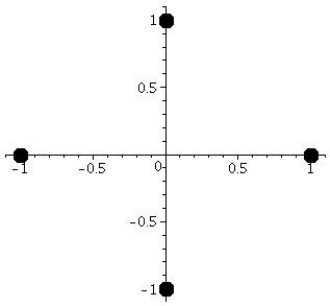
(A)



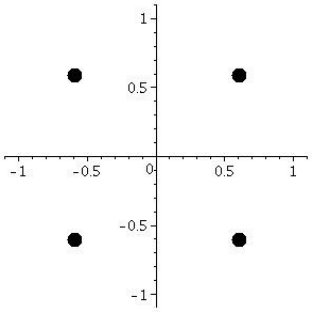
(C)



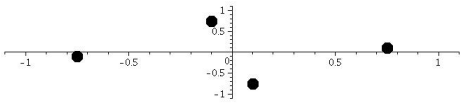
(E)



(B)



(D)



4. Consider the function

$$f(x)=\begin{cases}0, & \text{if } 0\le x\le 1\\ x, & \text{if } 1< x\le 2\end{cases}$$

defined on the interval $[0,2]$.

Let $\sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{2}\right)$ be a sine series series for $f(x)$ on the function $[0,2]$.

Using the same values of B_n for all x on the real line, define the function $g(x)=\sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{2}\right)$.

Find $g(-9)+g\left(\frac{-5}{2}\right)+g(6)$.

(A) $\frac{1}{2}$ (C) $\frac{3}{2}$ (E) 2 (G) $\frac{3}{4}$

(B) $-\frac{1}{2}$ (D) $\frac{3}{2}$ (F) 1 (H) none of these

Short Answer: This is a 5 part question that requires 5 solutions. Place your solution in the blank space provided.

5. Determine the value of the complex integral below for the various counterclockwise paths indicated

$$I=\frac{8}{\pi}\int_C \frac{dz}{z^4+10z^2+9}$$

a) If C is $|z-3i|=1$ _____

b) If C is $|z+i|=3$ _____

c) If C is $|z+2i|=2$ _____

d) If C is $|z-1-i|=2$ _____

e) If C is $|z+3|=2$ _____

Open ended: Show all work. A correct answer with little or no supporting work will receive little or no credit.

6. Expand $\dfrac{1}{z^2-7z+10}$ in a Laurent series valid for the indicated annular domain.

a) $2<|z|<5$ (center at $z=0$)

b) $0<|z-5|<3$ (center at $z=5$)

7. Consider the Sturm-Liouville problem $y''+\lambda y=0$ subject to $y(0)=0$ and $y'(3)=0$. Find the eigenfunctions $y(x)$ and the eigenvalues λ.

8. Solve $u_{xx}+u_{yy}=0,$ for a function $u(x,y)$ on $0\le x\le 1, 0\le y\le 2$ subject to the conditions $u(0,y)=0, \dfrac{\partial u}{\partial x}(1,y)=0, u(x,0)=0,$ and $u(x,2)=5\sin\left(\dfrac{3\pi x}{2}\right)-3\sin\left(\dfrac{7\pi x}{2}\right)$.

9. a) Solve $u_t=u_x$ using separation of variables.

b) Find two linearly independent solutions to $u_t=u_x+3$.
(There might be more than one way to represent the solution)

10. Let $\alpha=\displaystyle\int_{C_1}\left(2\bar{z}-z\right)dz$ where C_1 is defined by $x=-\dfrac{t}{2}, y=\dfrac{1}{2}t^2$ on $0\le t\le 2$.

Let $\beta=\displaystyle\int_{C_2}|z|^2\,dz$ where C_2 is defined by $x=t, y=\dfrac{1}{t}$ on $1\le t\le 2$.

Find $\alpha+\beta$.

ANSWERS:

1. B

2. E

3. C

4. F

5. a) -1/3 b) 1/3 c) -2/3 d) 1 e) 0

6.

$a)\ \ \frac{-1}{3}\sum_{n=0}^{\infty}\left[\frac{z^n}{5^{n+1}}+\frac{2^n}{z^{n+1}}\right]$ or $\frac{-1}{3}\left(\cdots+\frac{2^2}{z^3}+\frac{2}{z^2}+\frac{1}{z}+\frac{1}{5}+\frac{z}{5^2}+\frac{z^2}{5^3}+\cdots\right)$

$b)\ \ \sum_{n=0}^{\infty}\frac{(-1)^n(z-5)^{n-1}}{3^{n+1}}$ or $\frac{1}{3(z-5)}-\frac{1}{3^2}+\frac{(z-5)}{3^3}-\frac{(z-5)^2}{3^4}+\cdots$

7.

$y=C\sin\left((2n-1)\frac{\pi}{2}x\right),\ \ \lambda=\left((2n-1)\frac{\pi}{2}\right)^2\ \ n=1,2,3,\ldots$

8.

$$u(x,y)=\frac{5}{\sinh(3\pi)}\sin\left(\frac{3\pi}{2}x\right)\sinh\left(\frac{3\pi}{2}y\right)-\frac{3}{\sinh(7\pi)}\sin\left(\frac{7\pi}{2}x\right)\sinh\left(\frac{7\pi}{2}y\right)$$

9.

$a)\ u(x,t)=c_1e^{-\lambda(x+t)}+c_2$

$b)\ u_1(x,t)=e^{-\lambda(x+t)}+3t,\ \text{ and } u_2(x,t)=e^{-\lambda(x+t)}-3x$

10.

$\alpha=\frac{13}{2}+\frac{2}{3}i\ \text{ and }\beta=\frac{17}{6}-\frac{31}{24}i\ \Rightarrow\ \alpha+\beta=\frac{28}{3}-\frac{5}{8}i$