Math 241 Makeup Exam
Fall 2007

Name ______________________

- **Write your name in the space provided. Be prepared to show your Penn ID if asked.**
- **Indicate which professor's class you are enrolled in.**
- **You will have 2 hours to complete the exam.**
- **No calculators or books are permitted, however you may use an 8 ½ in. ✕ 11 in. sheet of notes.**
- **Mark your answer on this sheet and on the problem itself. Do not detach this sheet.**
- **Although this is a multiple choice exam, you must provide work to support your answer.**
- **Use the space provided beneath each problem to show your work.**
- **A correct answer with no work will be graded as an incorrect answer.**
- **Each problem is equally weighted with no partial credit given for skipped or incorrect solutions.**

1.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	Ⓗ
2.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	Ⓗ
3.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	Ⓗ
4.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	Ⓗ
5.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	Ⓗ
6.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	Ⓗ

7.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	Ⓗ
8.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	Ⓗ
9.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	Ⓗ
10.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	Ⓗ
11.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	Ⓗ
12.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	Ⓗ

1. Find the radius of convergence for the Taylor series of $f(z)=\dfrac{1}{z^8-1}$ about the point $z=2\sqrt{2}+2\sqrt{2}i$.

(A) 1 (C) 2 (E) 3 (G) 4

(B) $\dfrac{3}{2}$ (D) $\dfrac{5}{2}$ (F) $\dfrac{7}{2}$ (H) ∞

2. Consider the Laurent series for the function

$$f(z) = \frac{z^2 - 2z + 3}{z - 2}$$

in the region $|z-1| > 1$. What is the coefficient of the $(z-1)^{-2}$ term?

(A) −6	(C) −3	(E) 1	(G) 3
(B) −4	(D) 0	(F) 2	(H) 6

3. Find the constant k such that the function $v(x,y)=3x^2y+ky^3-x+1$ is a harmonic conjugate of the function $u(x,y)=x^3-3xy^2+y$.

(A) -3	(C) -1	(E) 1	(G) 3
(B) -2	(D) 0	(F) 2	(H) 4

4. Evaluate

$$\int_0^{2i} e^{iz}\,dz \, .$$

(A) $i\left(1-e^{-2}\right)$ (C) $1-ie^{-2}$ (E) $i-e^{-2}$ (G) $1-e^{-2}$

(B) $i\left(1+e^{-2}\right)$ (D) $1+ie^{-2}$ (F) $i+e^{-2}$ (H) $1+e^{-2}$

5. Evaluate

$$\frac{i}{2} \oint_{|z|=1} \frac{\left(z^2-1\right)^2}{z^2\left(z+\frac{1}{2}\right)\left(z+2\right)}\, dz$$

(A) $\frac{\pi}{10}$ (C) $\frac{\pi}{6}$ (E) $\frac{\pi}{3}$ (G) 1

(B) $\frac{\pi}{8}$ (D) $\frac{\pi}{4}$ (F) $\frac{\pi}{2}$ (H) π

6. Evaluate

$$\int_0^{2\pi} \frac{d\theta}{2-\cos\theta}.$$

(A) π

(B) $\frac{2\pi}{\sqrt{3}}$

(C) $\frac{2\pi}{3}$

(D) $\frac{\pi}{2}$

(E) $\frac{\pi}{\sqrt{3}}$

(F) $\frac{\pi}{3}$

(G) $\frac{\pi}{4}$

(H) $\frac{\pi}{6}$

7. Evaluate

$$\int_{-\infty}^{\infty} \frac{dx}{x^2-6x+13}.$$

(A) $-\pi$

(B) $-\dfrac{\pi}{12}$

(C) $-\dfrac{\pi}{24}$

(D) 0

(E) $\dfrac{\pi}{24}$

(F) $\dfrac{\pi}{12}$

(G) $\dfrac{\pi}{2}$

(H) π

8. In the Fourier series expansion of $f(x) = 2x^2 - 1$ on (-1,1) find the coefficient on the $\cos(4\pi x)$ term.

(A) 0

(B) $\frac{1}{2\pi^2}$

(C) $\frac{1}{2\pi}$

(D) $\frac{2}{\pi^2}$

(E) $\frac{1}{2}$

(F) $\frac{2}{\pi}$

(G) 1

(H) 2

9. Consider the Sturm-Liouville problem defined on $0 \le x \le \frac{\pi}{2}$:

$$y'' + \lambda y = 0 \qquad y(0) = 0,\ y'\left(\frac{\pi}{2}\right) = 0 \ .$$

Find all eigenvalues λ_n, $n = 0, 1, 2, \ldots$.

(A) $\lambda_n = n^2$ (C) $\lambda_n = \frac{n}{4}$ (E) $\lambda_n = \frac{(2n-1)\pi}{4}$ (G) $\lambda_n = \frac{(2n-1)^2 \pi}{2}$

(B) $\lambda_n = \frac{n^2}{4}$ (D) $\lambda_n = \frac{(2n-1)\pi}{2}$ (F) $\lambda_n = (2n-1)^2$ (H) $\lambda_n = \frac{(2n-1)^2 \pi}{4}$

10. The solution $u(x,t)$ defined for $0 \le x \le 2, t \ge 0$ to the wave equation

$u_{tt} = u_{xx}$ with boundary conditions $u_x(0,t) = u_x(2,t) = 0$ is

$$u(x,t) = \sum_{n=0}^{\infty} \left[A_n \cos\left(\tfrac{n\pi}{2} t\right) + B_n \sin\left(\tfrac{n\pi}{2} t\right) \right] \cos\left(\tfrac{n\pi}{2} x\right).$$

Find $u\left(\frac{1}{3}, \frac{1}{2}\right)$ with initial conditions $u(x,0) = 3\cos(\pi x)$ and $u_t(x,0) = 2\cos(3\pi x)$.

(A) $\frac{1}{\pi}$ (C) $\frac{3}{\pi}$ (E) $\frac{2}{3\pi}$ (G) $\frac{1}{2}$

(B) $\frac{2}{\pi}$ (D) $\frac{1}{3\pi}$ (F) $\frac{1}{3}$ (H) 2

11. Let $u(x,t)$ be a function defined for $0 \le x \le \pi, t \ge 0$ such that

$$u_t = u_{xx} + 2u_x$$

with boundary conditions $u(0,t) = u(\pi,t) = 0$ for all $t \ge 0$

and initial condition $u(x,0) = e^{-x}\sin(2x)$.

Use separation of variables to find $u\left(\frac{\pi}{4},1\right)$.

(With separation constant $-\lambda$, you will find non-trivial solutions when $\lambda > 1$, say $\lambda = 1+\alpha^2$)

(A) $e^{-1-\frac{\pi}{2}}$	(C) $e^{-5-\frac{\pi}{2}}$	(E) $e^{-1-\frac{\pi}{4}}$	(G) $e^{-5-\frac{\pi}{4}}$
(B) $e^{-2-\frac{\pi}{2}}$	(D) $e^{-10-\frac{\pi}{2}}$	(F) $e^{-2-\frac{\pi}{4}}$	(H) $e^{-10-\frac{\pi}{4}}$

12. Consider a circular plate of radius 1 whose circular edge is maintained at the temperature $u(1,\theta)=\theta$. The steady-state temperature $u(r,\theta)=A_0+\sum_{n=1}^{\infty} r^n\left[A_n\cos(n\theta)+B_n\sin(n\theta)\right]$ satisfies $u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta}=0$. Find the coefficient of the $\sin(3\theta)$ term.

(A) 0
(B) $\frac{1}{3}$
(C) $\frac{2}{3}$
(D) $\frac{-2}{3}$
(E) $\frac{\pi}{3}$
(F) $\frac{-\pi}{3}$
(G) $\frac{2\pi}{3}$
(H) $\frac{-2\pi}{3}$