1. Find the Fourier series for the function $f(x)=\begin{cases}-1-x, & -1<x<0\\ 1-x, & 0<x<1\end{cases}$. Write out terms through $n = 3$.

2. Solve the one-dimensional heat equation for a laterally insulated bar of length π whose ends are held at 0° and across which the initial temperature distribution is given by $f(x)=\begin{cases}x, & < x < \dfrac{\pi}{2}\\ \pi - x, & \dfrac{\pi}{2} < x < \pi\end{cases}$.

3. Find *all* product solutions for $u(x, y)$ if $xu_x - yu_y = 0$.

4. Find all of the cube roots of $z = 4 + i4\sqrt{3}$. Sketch the roots iin the complex plane.

5. Which, if any of the functions listed below are analytic in the domain $z \neq 0$?
   a) $\log z$ b) $\dfrac{1}{z^2}$ c) $\dfrac{1}{e^z}$

6. Is there a function $v(x, y)$ corresponding to $u(x,y) = x^3 - y^3 + 6xy$ such that $f(z) = u(x,y) + iv(x,y)$ is an analytic complex function? If such a v exists, find it, write $f(z)$ and compute $f'(z)$.

7. Evaluate $\int_C z\bar{z}\, dz$ if C is the straight line from $z = 0$ to $z = 2+4i$.

8. Evaluate $\int_C \sin z\, dz$ where C is the semi-circle $|z| = 1$ in the upper half-plane starting at $z = 1$ and ending at $z = -1$.

9. Evaluate $\oint_C \dfrac{7z-6}{z^2-2z} dz$ where C is the ellipse $\dfrac{(x-1)^2}{9} + y^2 = 1$.

10. Find the Maclaurin series for $f(z) = \arctan(z)$ and determine the radius of convergence of your series by some suitable means. {HINT: first find a series for the derivative $f'(z) = \dfrac{1}{z^2+1}$ then integrate that series termwise.}

11. Write out the first four terms of the Laurent series for $f(z) = z^2 \sin\left(\dfrac{1}{z}\right)$ with center $z_0 = 0$, then:
    a) give the residue of $f(z)$ at $z = 0$;
    b) check one-- $z_0 = 0$ is ❑ a pole of order ____________________;
    ❑ an essential singularity.

12. Evaluate the contour integral $\oint_C z^3 e^{-1/z^2}\,dz$ where C is any circle centered at the $z = 0$ traversed in the clockwise direction.

13. Evaluate the real integral $\int_0^{2\pi} \frac{1}{1+\cos\theta}\,d\theta$.

14. Evaluate the contour integral $\oint_C \frac{e^z}{z(z^2+\pi^2)}\,dz$ over each of the following (positively oriented) contours:

a0 $|z| = 4$ b) $|z - 4i| = 2$ c) $|z - 2| = 1$