Final Exam - April 29, 2005
Math 241

Name:____

Instructor:____

TA and Recitation:____

No calculators, books or notes are allowed. You may use two $8.5'' \times 11''$ two-sided pages of crib sheets, which must be turned in with your exam. Show your work in the space provided for each problem, or in the back of the corresponding page, or in one of the blank pages provided in the back of the exam.

Make sure to show all your work, clearly and in order, if you want to get full credit. We reserve the right to take off points if we cannot see how you arrived at your answer (even if your final answer is correct). Good luck!

Question	Points
1	/10
2	/10
3	/10
4	/10
5	/10
6	/10
7	/10
8	/14
9	/16
TOTAL	/100

1. TRUE or FALSE.

(a) The radius of convergence of the Taylor series of $f(z) = \dfrac{\sin z}{(z+2)^2}$ around $z = 1$ is R=1.

(b) Let C_1 be the positively oriented circle of radius 2 and let C_2 be the positively oriented circle of radius 5. Then

$$\oint_{C_1} \left[\frac{1}{z+1} + \frac{1}{z-4}\right] dz = \oint_{C_2} \left[\frac{1}{z+1} + \frac{1}{z-4}\right] dz$$

2. **(I)** If $f(x) = x^2 - x$ is expanded in a Fourier series on the interval $[-2, 2]$, then at $x = 2$ the series will converge to

(a) 0

(b) 1

(b) 2

(c) 3

(d) 4

(e) 5

(f) 6

(II) What is the image of the line $x = 1$ under the map $f(z) = e^z$?

(a) A circle

(b) A horizontal line

(c) A vertical line

(d) A hyperbola

(e) A logarithmic spiral

(f) None of these

3. Compute the complex Fourier series for $f(x) = \begin{cases} 3, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ which is valid on the interval $(-2, 2)$.

4. Using separation of variables find the solution of

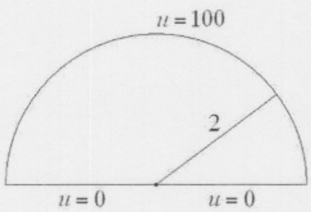
$$\frac{\partial^2 u}{\partial t^2} - 2\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

satisfying

$$u(0,t) = 0, \qquad u\left(\frac{\pi}{2}, t\right) = 0,$$
$$u(x,0) = 0, \qquad u_t(x,0) = f(x)$$

Express the coefficients of the solution in terms of $f(x)$.

5. Find the steady-state temperature in a semicircular metal plate of radius 2, if the round part is held at a temperature of 100 degrees and the flat bottom part is held at 0 degrees.



6. Using a Fourier Transform find the steady-state temperature $u(x, y)$ on the semi-infinite plate satisfying

$$\begin{aligned} u_{xx} + u_{yy} &= 0, &\quad& 0 < x < \pi, y > 0 \\ u_y(x, 0) &= 0, && 0 < x < \pi \\ u(0, y) &= 0, && u(\pi, y) = \begin{cases} 1 & y < 1 \\ 0 & y > 1 \end{cases} \end{aligned}$$

Express your final answer as an inverse Fourier transform integral.

7. Find the Laurent series of $f(z) = \dfrac{2z}{z^2 - 2z - 8}$ valid for $|z+2| > 6$.

8. Classify the singularity of each of the following functions at $z_0 = 0$ and find the corresponding residue.

   (a) $f(z) = \dfrac{1}{z(e^z - 1)}$ Type: Residue=

   (b) $f(z) = z^3 e^{-1/z^2}$ Type: Residue=

9. Compute the following integrals:

(a) $\displaystyle\int_0^{2\pi} \frac{1}{5 + 4\cos\theta}\, d\theta$

(b) $\displaystyle\int_{-\infty}^{\infty} \frac{\cos x}{(x^2+4)(x^2+1)}\,dx$

Feel free to use this page for scratch.