Math 241 Fall 05 Final Exam B

Instructions:. Do four of five in Part One and six of the seven in Part Two. If you try more than the required number , only your best will be counted. You may use one standard size 8.5 by 11 inch "cheat sheet" and your calculator. Turn in your exam book **and this question sheet** but not your "cheat sheet".

Part One (Ten points each: Do four)

1. Find the Fourier series for the function which is 2π periodic and whose value for $0 \le x < 2\pi$ is $|x|$

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2. Use the Fourier sine transform to solve the following version of the heat equation :

$\dfrac{\partial u}{\partial t} = k\,\dfrac{\partial^2 u}{\partial x^2}$ $\quad t > 0,\ x > 0$ subject to $u(0,t) = 0$, $u(x,0) = e^{-x}$. Use the fact (easy to prove and similar to Laplace transform) that $\mathcal{F}_s\left[e^{-x}\right] = \dfrac{\alpha}{1+\alpha^2}$

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3. Solve the Sturm - Liouville problem $y'' + \lambda y = 0$ subject to $y'(0) = 0$ and $y'(\pi) = 0$ on the interval $[0, \pi]$. Be sure to deliver ALL eigenvalues and ALL eigenfunctions and to demonstrate that you have found ALL such

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4. Solve the (boundaryless) wave equation $\dfrac{\partial^2 u}{\partial t^2} = c^2\,\dfrac{\partial^2 u}{\partial x^2}$ $\quad -\infty < x < \infty,\ t > 0$ subject to $u(x,0) = f(x)$ (initial deflection) and $\dfrac{\partial u}{\partial t}(x, 0) = 0$ (initial speed zero). For best results use the full Fourier transform. Your answer will of course have to depend on $f(x)$.

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5. Solve Laplacee' s equation $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$, $0 \le x \le a$, $0 \le y \le b$ subject to

$u(x,0) = 0$, $u(x,b) = 100$, $u(0,y) = 0$, $u(a,y) = 0$

Part Two (Ten points each; do six)

6. Let $f(z)=z^2+1$. Let Γ be the path parametrized by $z(t)=(t+1)+i(t^2+2t+1)$ $0\le t\le 1$ Compute , by any legitimate method of your choice , $\int_\Gamma f(z)dz$

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7. For $f(z)=\dfrac{1}{z^2(z-3)}$. compute the Laurent series valid for $0<|z|<3$

Use your result to compute Residue$(f(z), z=0)$

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8. The function $f(z)=u(x,y)+i\,v(x,y)$ is analytic. If $u(x,y)=x^3-3xy^2$ then

(i) Compute $v(x,y)$ (ii) compute $f'(z)$

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9. Compute $\oint_C \dfrac{ze^z}{(z-2)^2}dz$ where C is the circle $|z|=3$

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10. Compute $\displaystyle\int_{-\infty}^{\infty}\frac{x^2dx}{(x^2+1)^2(x^2+2x+2)}$

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11. For $f(z)=\dfrac{z^2-2z}{(z+1)^2(z^2+4)}$. find the residue of $f(z)$ at the singular point $z=-1$.

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12. Compute $\displaystyle\int_0^{2\pi}\frac{d\theta}{13-5\sin\theta}$