Mathematics 241 Makeup Final Examination

Tuesday, January 13, 2004

Your name, printed: ....

Your signature: ....

Your Penn student ID#: ....

Circle the name of your Professor: Shatz Wilf

This is a closed book exam. You may use during the exam, a single 5x7 inch card on which you may have written on both sides, if you wish. No more than one such card, and no larger size, please. No other books, papers, calculators, or other materials may be consulted during an exam.

The exam consists of twenty (20) multiple choice questions, and each question offers a choice of five or six answers. Circle the answer that you choose, in each case, on the question sheet itself. The exam booklets are given to you for your convenience in working out the problems, but do not hand in these booklets when the exam is over. Hand in only the exam itself, with your answers to the questions circled.

For grader's use only:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Do not turn the page and start work until the proctors announce the start of the exam.

1. While we are solving a certain heat conduction problem, suppose that the solution which satisfies $u(x,0) = 1$ is

$$u(x,t) = \sum_{n=1}^{\infty} A_n e^{-kn^2\pi^2 t/L^2} \sin\frac{n\pi x}{L}.$$

Then which one of the following is true:

   (a) $u_x(0,t) = u_x(L,t) = 0$
   (b) $u_x(0,t) = u_x(L,t)$
   (c) $u_x(0,t) = u(0,t) = 0$
   (d) $u(0,t) = u(L,t) = 0$
   (e) $u_x(L,t) = u(L,t) = 0$

2. The residue of $ze^{1/(2z)}$ at the origin is

   (a) $1/2$
   (b) $2\pi i$
   (c) $1/8$
   (d) πi
   (e) none of the above

3. Given that k is a constant and that $x^2 - ky^2$ is the real part of an analytic function, then

   (a) $k > 0$
   (b) $k = 0$
   (c) $k = 1$
   (d) no value of k can do this
   (e) $k = -1$

4. The integral of $e^z/(z-1)^3$ counterclockwise around a small circle centered at $z_0 = 1$ is

   (a) $2\pi i$
   (b) $\pi i/12$
   (c) $\pi i/3$
   (d) $e\pi i$

(e) none of the above

5. Which one of the following is true:

   (a) -6 has five fifth roots, and none of them is a real number

   (b) -1 has three cube roots, and two of them are real

   (c) 2 has 12 12th roots and one of them is real

   (d) 1 has n nth roots, and for every n, exactly one of them is real.

   (e) if $a + ib$ is an nth root of 1, then so is $a - ib$.

6. In polar coordinates, Laplace's equation for the temperature $u(r, \theta)$ in a circular plate centered at the origin is $u_{rr} + u_r/r + u_{\theta\theta}/r^2 = 0$. Look for a product solution $u(r, \theta) = R(r)\Theta(\theta)$, and name the separation constant so that the Θ differential equation turns out to be $\Theta''(\theta) + \lambda^2\Theta(\theta) = 0$. Then which one of the following is true:

   (a) the eigenvalues λ of the problem are the nonnegative integers and when $\lambda = 0$, $\Theta(\theta)$ is a nonconstant linear function of θ.

   (b) the eigenvalues λ of the problem are the positive integers

   (c) the eigenvalues λ of the problem are the nonnegative integers and when $\lambda = 0$, $\Theta(\theta)$ is a constant.

   (d) the eigenvalues λ of the problem are the nonnegative integers and when $\lambda = 0$, $R(r)$ is of the form $c_1 + c_2 \ln r$ where neither c_1 nor c_2 is necessarily 0.

   (e) none of the above are true

7. A thin rod of length π starts out at a temperature $f(x) = 100 \sin x$. If its ends are kept always at 0, and units are chosen so that the PDE is $u_t = u_{xx}$, then after 1 unit of time has elapsed the temperature of the rod one-quarter of the way from either end is

   (a) $100\sqrt{2}/e$

   (b) $100e$

   (c) $50/e$

   (d) $50\sqrt{2}/e$

   (e) $50e\sqrt{2}$

8. The center, z_0, and the radius, R, of the circle of convergence of the power series

$$\sum_{k=0}^{\infty} \frac{1}{k^2} \left(\frac{3+4i}{3-4i} \right)^k (z-3)^k$$

(a) are $z_0 = 3$ and $R = \sqrt{2}$
(b) are $z_0 = 0$ and $R = \sqrt{2}$
(c) are $z_0 = 3$ and $R = 1$
(d) are $z_0 = 0$ and $R = \sqrt{2}$
(e) are none of the above

9. The value of $\int_0^{2\pi} dt/(10 - 6\cos t)$ is

(a) $-1/8$
(b) $\pi/4$
(c) $1/2$
(d) $\pi/2$
(e) none of the above

10. The value of $\int_{-\infty}^{\infty} dx/(x^2 - 6x + 25)$ is

(a) $\pi/4$
(b) $1/8$
(c) $\pi/8$
(d) $1/4$
(e) none of the above

11. In the Frobenius series solution $y(x) = \sum_{n=0}^{\infty} a_n x^{n+r}$ of the differential equation $2xy'' + (1+x)y' + y = 0$, the two possible values of the index r are

(a) 0 and 1
(b) 0 and 2
(c) 0 and 1/2
(d) 1 and 2
(e) none of the above

12. Which of the following functions has a pole of order 2 and a pole of order 1 and a removable singularity?

    (a) $(e^z - 1)/(z^2(z-1))$
    (b) $(e^z - 1)/(z(z^2-9)(z+3))$
    (c) $\sin z/(z(z+1)(z-1))$
    (d) $\sin z/(z^2(z-1))$
    (e) none of the above

13. The value of the integral $(1/(2\pi i)) \int_\gamma \operatorname{Re}(z) dz$, where γ is the unit circle $|z| = 1$ traversed counterclockwise, is

    (a) 0
    (b) $\frac{1}{2}$
    (c) 1
    (d) $\frac{3}{2}$
    (e) 2

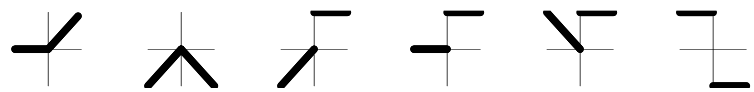
14. The Laurent series of the function $1/(z(z-1)^2)$ valid in the annulus $0 < |z-1| < 1$ is

    (a) $\frac{1}{(z-1)^2} + \frac{1}{z-1} + 1 - (z-1) + (z-1)^2 - \ldots$
    (b) $\frac{-1}{(z-1)^2} + \frac{1}{z-1} + 1 - (z-1) + (z-1)^2 - \ldots$
    (c) $\frac{1}{(z-1)^2} - \frac{1}{z-1} + 1 - (z-1) + (z-1)^2 - \ldots$
    (d) $\frac{1}{(z-1)^2} + \frac{1}{z-1} + 1 + (z-1) + (z-1)^2 + \ldots$
    (e) $\frac{-1}{(z-1)^2} + \frac{1}{z-1} + 1 + (z-1) + (z-1)^2 + \ldots$

15. For the differential equation $x^2 y'' + (x-1)y = 0$, the result of using the method of Frobenius is

    (a) Two solutions, both of which take finite values at $x = 0$
    (b) Two solutions, neither of which takes finite values at $x = 0$
    (c) An indicial equation whose roots differ by an integer
    (d) Two solutions, only one of which takes finite values at $x = 0$
    (e) No Frobenius-type solutions exist.

16. A certain function $f(x)$ has a Fourier series of the form $\sum_{k=0}^{\infty} A_{2k} \cos(2kx)$. Of the six graphs below, only one might be the graph of this $f(x)$. Which one? (Just circle the number underneath the graph that you choose, to indicate your answer.)



1 2 3 4 5 6

17. Consider the complex number $w = e^{z \log i}$, where z is an unknown complex number. If we want every value of w to lie on the unit circle, then z must

(a) have real part $= 0$

(b) be real

(c) lie in the lower half plane

(d) lie in the left half plane

(e) lie in the upper half plane

(f) lie in the right half plane

18. A certain odd function $f(x)$, periodic of period $2L$, has the Fourier series $f(x) = \sum_{k=1}^{\infty} b_k \sin(k\pi x/L)$. A string of length L is fixed at its endpoints and has initial displacement $f(x)$, and initial velocity $g(x) = 0$. Then the subsequent motion of the string is described by which one of the following

(a) $\sum_{k=1}^{\infty} \sin(k\pi x/L) \cos(k\pi ct/L)$

(b) $\sum_{k=1}^{\infty} b_k \sin(k\pi x/L) \cos(kct)$

(c) $\sum_{k=1}^{\infty} b_k \sin(k\pi x/L) \sin(k\pi ct/L)$

(d) $\sum_{k=1}^{\infty} b_k \cos(k\pi x/L) \cos(k\pi ct/L)$

(e) $\sum_{k=1}^{\infty} b_k \sin(k\pi x/L) \cos(k\pi ct/L)$

(f) $\sum_{k=1}^{\infty} b_k \cos(k\pi x/L) \sin(k\pi ct/L)$

19. Consider the boundary value problem on a square of side length 1, for Laplace's equation $u_{xx} + u_{yy} = 0$, in which $u(0, y) = 0$ and $u(1, y) = 0$ and $u(x, 0) = 0$ and $u(x, 1) = \sin(2\pi x)$. Then $u(1/4, 1/2)$ is

(a) $(e^{\pi} - e^{-\pi})/(e^{2\pi} - e^{-2\pi})$

(b) $(e^{\pi} + e^{-\pi})/(e^{2\pi} - e^{-2\pi})$

(c) $(e^{\pi} - e^{-\pi})/(e^{2\pi} + e^{-2\pi})$

(d) $(e^{\pi} + e^{-\pi})/(e^{2\pi} + e^{-2\pi})$

(e) $(e^{2\pi} - 1)/(e^{4\pi} - 1)$

(f) $(e^{2\pi} + 1)/(e^{4\pi} + 1)$

20. The differential equation $y'' + xy' + y = 0$ has a series solution $y = 1 + a_2x^2 + a_3x^3 + \ldots$. Then

(a) all a_i's in which i is even and ≥ 4 are 0.

(b) $a_2 + a_4 > 0$

(c) all a_i's in which i is odd and ≥ 3 are 0.

(d) The absolute values of the a's with even subscripts decrease steadily, and a_6 is the first one with absolute value $< 1/100$.

(e) $a_2 + a_3 + a_4 + a_5 + a_6 > 0$.

(f) $y'(0) = 1$