Mathematics 241–Syllabus and Core Problems

Math 241. Calculus, Part IV. Staff. Prerequisite(s): MATH 114 and 240. Sturm-Liouville problems, orthogonal functions, Fourier series, and partial differential equations including solutions of the wave, heat and Laplace equations, Fourier transforms.

Text: Haberman, Richard *Applied Partial Differential Equations with Fourier Series and Boundary Value Problems, 5th Edition* ©2012, Pearson/Prentice Hall, Publishers

Review of Prerequisite Concepts (0.5 weeks)

Vector calculus: partial and directional derivatives, gradients, the Laplacian. the divergence theorem
Complex variables: arithmetic, basic transcendental functions, Euler's formula

Introduction to the Heat and Laplace Equations (1.5 weeks)

Chapter 1: Heat Equation		**Core Problems**
(1.2)	Derivation of the Conduction of Heat in a 1D Rod	1, 2, 3, 4
(1.3)	Boundary Conditions	1, 2, 3
(1.4)	Equilibrium Temperature Distribution	1, 4, 7, 10
	(1.4.1) Prescribed Temperature	
	(1.4.2) Insulated Boundaries	
(1.5)	(*optional*) Derivation of the Heat Equation in 2D & 3D	1, 8, 9
	Polar and Cylindrical Coordinates	3

Separation of Variables (2 weeks)

Chapter 2: Method of Separation of Variables		
(2.2)	Linearity	2, 4, 5
(2.3)	Heat Equation with Zero Temp. at Finite Ends	1, 2, 3, 7, 8, 11
	(2.3.2) Separation of Variables	
	(2.3.3) Time-Dependent Equation	
	(2.3.4) Boundary Value Problem	
	(2.3.5) Product Solutions and Superposition	
	(2.3.6) Orthogonality of Sines	
(2.4)	Worked Examples	1, 2, 4, 7
	(2.4.1) Heat Conduction in a Rod with Insulated Ends	
	(2.4.2) Thin circular rod	
	(2.4.3) Summary	
(2.5)	Laplace's Equation	1, 2, 3, 5
	(2.5.1) Laplace's Equation in a Rectangle	
	(2.5.2) Laplace's Equation in a Disk	
	(2.5.4) (*optional*) Qualitative Properties of Laplace's Equation	

Theoretical Issues Concerning Fourier Series (1 week)

Chapter 3: Fourier Series

(3.2)	Convergence Theorem for Fourier Series	1, 2
(3.3)	Fourier Cosine and Sine Series	1, 2, 18
	(3.3.1) Fourier Sine Series	
	(3.3.2) Fourier Cosine Series	
	(3.3.3) Combined Sine and Cosine Series	
	(3.3.4) Continuity	
(3.4)	(*optional*) Term-by-Term Differentiation of Fourier Series	1, 5, 6
(3.5)	(*optional*) Term-by-Term Integration of Fourier Series	2, 4
(3.6)	Complex Form of Fourier Series	1, 2

Introduction to the Wave Equation (1 week)

Chapter 4: Wave Equation: Vibrating String and Membrane

(4.2)	Derivation of a Vertically Vibrating String	1, 5
(4.3)	Boundary Conditions	1, 2
(4.4)	Vibrating String with Fixed Ends	1, 2, 3, 9, 10
(4.5)	Vibrating Membrane	1
(4.6)	(*optional*) Reflection and Refraction of Waves	1, 3, 4

Sturm-Liouville Theory (1 week)

Chapter 5: Sturm-Liouville Eigenvalue Problems

(5.3)	Sturm-Liouville Eigenvalue Problems	2, 5, 6
	(5.3.1) General Classification	
	(5.3.2) Regular Sturm-Liouville Eigenvalue Problem	
	(5.3.3) Example and Illustration of Theorems	
(5.4)	Worked Example: Heat Flow in a Nonuniform Rod	2, 3
(5.9)	Large Eigenvalues (Asymptotic Behavior)	1, 2
(5.10)	Approximation Properties	1, 4

PDEs in Higher Dimensions (2.5 weeks)

Supplementary Topic: Power Series Solutions of ODEs and the Method of Frobenius

Chapter 7: Higher Dimensional PDEs

(7.2)	Separation of the Time Variable	
	(7.2.1) Vibrating Membrane: Any Shape	
	(7.2.2) Heat Conduction: Any Region	
	(7.2.3) Summary	
(7.3)	Vibrating Rectangular Membrane	1, 2, 4, 5
(7.4)	Statements and Illustrations of Theorems for the Eigenvalue Problem $\nabla^2\phi + \lambda\phi = 0$	1
(7.7)	Vibrating Circular Membrane and Bessel Functions	1, 2, 6, 10
	(7.7.2) Separation of Variables	

	(7.7.3) Eigenvalue Problems (One Dimensional)	
	(7.7.4) Bessel's Differential Equation	
	(7.7.5) Singular Points and Bessel's Differential Equation	
	(7.7.6) Bessel Functions and Their Asymptotic Properties	
	(7.7.7) Eigenvalue Problem Involving Bessel Functions	
	(7.7.8) Initial Value Problem for a Vibrating Circular Membrane	
(7.8)	(*optional*) More on Bessel Functions	1, 2, 5, 11
(7.9)	Laplace's Equation on a Circular Cylinder	2, 3
	(7.9.2) Separation of Variables	
	(7.9.3) Zero Temperature on the Sides, Bottom, and Top	
	(7.9.4) Zero Temperature on the Top and Bottom	
	(7.9.5) (*optional*) Modified Bessel Functions	
(7.10)	Spherical Problems and Legendre Polynomials	1, 2, 10
	(7.10.2) Separation of Variables and 1D Eigenvalue Problems	
	(7.10.3) Associated Legendre Functions and Legendre Polynomials	
	(7.10.4) Radial Eigenvalue Problems	
	(7.10.5) Product Solutions, Modes of Vibration, and the IVP	
	(7.10.6) Laplace's Equation Inside a Spherical Cavity	

Nonhomogeneous Source Terms and Boundary Conditions (1.5 weeks)

Chapter 8: Nonhomogeneous Problems

(8.2)	Heat Flow with Nonhomogeneous Conditions	1, 3, 4
(8.3)	Method of Eigenfunction Expansion, Differentiation of Series	1, 2, 3, 6
(8.4)	(*optional*) Eigenfunction Expansion, Green's Formula	1, 4
(8.5)	Forced Vibrating Membranes and Resonance	1, 2, 3
(8.6)	Poisson's Equation	1, 3, 6

The Fourier Transform (1 week)

Chapter 10: Infinite Domain Problems

(10.2)	Heat Equation on an Infinite Domain	1, 2
(10.3)	Fourier Transform Pair	1, 2, 6, 7
	(10.3.1) Motivation from the Fourier Series Identity	
	(10.3.2) Fourier Transform	
	(10.3.3) Inverse Fourier Transform of a Gaussian	
(10.4)	Fourier Transform and the Heat Equation	3, 4, 7(a), 7(b)
	(10.4.1) Heat Equation	
	(10.4.2) Transforms of Derivatives	
	(10.4.3) Convolution Theorem	
(10.6)	Worked Examples Using Transforms	3, 9, 18
	(10.6.1) One-Dimensional Wave Equation on an Infinite Interval	
	(10.6.3) Laplace's Equation in a Half-Plane	
	(10.6.5) Heat Equation in a Plane	

Numerical Solutions of PDEs (1 week)

Chapter 6: Finite Difference Numerical Methods for PDEs

(6.2)	Finite Differences and Truncated Taylor Series	1, 5, 7
(6.3)	Heat Equation	1, 5, 16
	(6.3.1) Heat Equation	
	(6.3.2) A Partial Difference Equation	
	(6.3.3) Computations	
	(6.3.4) Fourier-von Neumann Stability Analysis	
	(6.3.9) Other Types of Boundary Conditions	
(6.5)	Wave Equation	1, 2, 5, 6