Math240, Spring 2007

Answer Key

1. (i) false, (ii) true, (iii) false.

2. Solution is not unique, a possible solution is as follows. Since $\det(A^T) = \det(A)$, we obtain that $\det(A) = \det(-A) = (-1)^3 \det(A) = -\det(A)$. So, $\det(A) = 0$, and therefore A is singular and its rank is smaller than 3.

3. Answer is not unique, a possible answer is

$$P = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix}, \; D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{pmatrix}.$$

4. (i) true, (ii) true, (iii) true.

5. $c_2 = 0$ and $c_{n+3} = \frac{2}{(n+3)(n+2)} c_n$ for $n \geq 0$; a possible choice of y's is as follows: $y_1 = 1 + \frac{1}{3}x^3 + \frac{1}{45}x^6 + \ldots$ and $y_2 = x + \frac{1}{6}x^4 + \frac{1}{126}x^7 + \ldots$ (a more tricky choice is to take, for example, $y_1 = 1 + x + \frac{1}{3}x^3 + \ldots$ and $y_2 = 1 - x + \frac{1}{3}x^3 + \ldots$).

6. $y_1 = 10e^{5t} + 6e^{-t}, y_2 = 5e^{5t} - 6e^{-t}$.

7. $y = \left(11e^{-3t} + 3\sin(t) - \cos(t)\right)/10$.

8. (g)

9. (f)

10. (g)

11. (c)

12. (c)

13. (b)

14. (h)

15. (e)