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Instructions.

Please write your name and Penn ID in the space provided above, and fill in the oval identifying your section and instructors. You will have two hours to complete this exam.

You are allowed to use one $8\frac{1}{2} \times 11$ sheet, both sides, for notes you wrote yourself. You are not allowed to use calculators.

Do not detach this sheet from the body of the exam.

This is a multiple-choice test, but you must show your work. Blind guessing will not be credited.

Please mark your answer on both the front cover and on the problem itself. If you change an answer, be absolutely clear which choice is your final answer.

All of the problems have exactly one correct answer.

All problems have equal weight. No partial credit will be given. No penalties for incorrect answers will be taken.

Questions 1-11						**Questions 12-22**					
1.	(A)	(B)	(C)	(D)	(E)	12.	(A)	(B)	(C)	(D)	(E)
2.	(A)	(B)	(C)	(D)	(E)	13.	(A)	(B)	(C)	(D)	(E)
3.	(A)	(B)	(C)	(D)	(E)	14.	(A)	(B)	(C)	(D)	(E)
4.	(A)	(B)	(C)	(D)	(E)	15.	(A)	(B)	(C)	(D)	(E)
5.	(A)	(B)	(C)	(D)	(E)	16.	(A)	(B)	(C)	(D)	(E)
6.	(A)	(B)	(C)	(D)	(E)	17.	(A)	(B)	(C)	(D)	(E)
7.	(A)	(B)	(C)	(D)	(E)	18.	(A)	(B)	(C)	(D)	(E)
8.	(A)	(B)	(C)	(D)	(E)	19.	(A)	(B)	(C)	(D)	(E)
9.	(A)	(B)	(C)	(D)	(E)	20.	(A)	(B)	(C)	(D)	(E)
10.	(A)	(B)	(C)	(D)	(E)	21.	(A)	(B)	(C)	(D)	(E)
11.	(A)	(B)	(C)	(D)	(E)	22.	(A)	(B)	(C)	(D)	(E)

1. For the matrices A^{-1} and B^{-1} below, find $(AB)^{-1}$.

$$A^{-1} = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} \quad B^{-1} = \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}$$

(A) $\begin{pmatrix} 8 & 12 \\ 12 & 5 \end{pmatrix}$ (B) $\begin{pmatrix} 8 & 3 \\ 12 & 5 \end{pmatrix}$ (C) $\begin{pmatrix} 5 & 6 \\ 6 & 8 \end{pmatrix}$ (D) $\begin{pmatrix} 5 & 8 \\ 8 & 6 \end{pmatrix}$

(E) This can't be done: one of A, B is singular, and AB is undefined.

2. Find $y(\pi)$, where y satisfies the differential equation

$$\frac{d^4y}{dx^4} - 16y = 0\,,$$

subject to the initial condition

$$y(0) = 1, \quad y'(0) = 0, \quad y''(0) = -4, \quad y'''(0) = 0\,.$$

As a reminder, $A^4 - 16 = (A^2 - 4)(A^2 + 4) = (A - 2)(A + 2)(A^2 + 4)$.

(A) 0 (B) 1 (C) $\frac{1}{2}e^{2\pi} + \frac{3}{2}$ (D) $\frac{1}{2}e^{2\pi} + \frac{1}{2}e^{-2\pi} - \frac{1}{2}$ (E) $\frac{1}{2}e^{2\pi} - \frac{1}{2}e^{-2\pi} + \frac{3}{2}$

3. Give the general solution to the differential equation

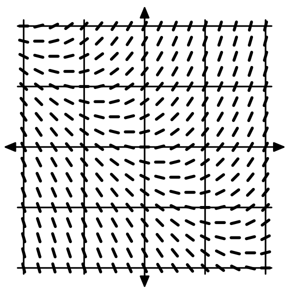
$$x^2y'' - 2xy' + 2y = 0\,.$$

(A) $y = c_1x^2 + c_2x^3$ (B) $y = c_1 + c_2x^{-2}$ (C) $y = c_1x + c_2x^3$
(D) $y = c_1 + c_2x^2$ (E) $y = c_1x + c_2x^2$

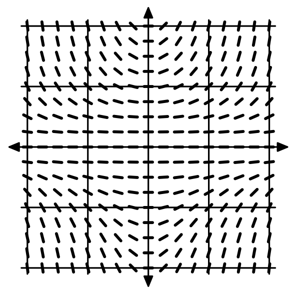
4. Which of the following diagrams is the direction field for the differential equation $y' = xy$?

In the figures, the x-axis and the y-axis both run from -2 to 2.

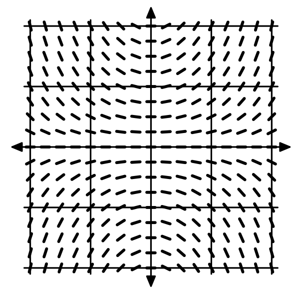
(A)



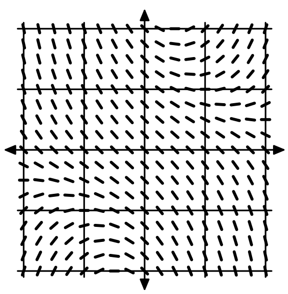
(B)



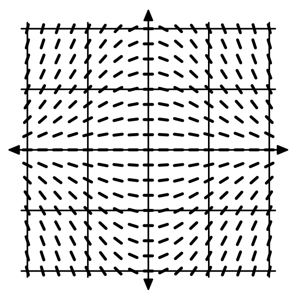
(C)



(D)



(E)



5. Evaluate the inverse Laplace transform

$$\mathcal{L}^{-1}\left\{\frac{4s+23}{s^2+4s+29}\right\}.$$

(A) $2e^{4t}\sin 5t + 3e^{4t}\cos 5t$

(B) $3e^{2t}\sin 5t - 2e^{2t}\cos 5t$

(C) $2e^{-2t}\sin 5t + 3e^{-2t}\cos 5t$

(D) $3e^{-2t}\sin 5t + 4e^{-2t}\cos 5t$

(E) $2e^{2t}\sin 5t + 2e^{2t}\cos 5t$

6. The differential equation

$$y' - 2y = e^{2t}\,,$$

with initial condition $y(0) = 3$, has its solution expressible using inverse Laplace transforms as:

(A) $\mathcal{L}^{-1}\left\{\dfrac{3}{s-2}\right\} + \mathcal{L}^{-1}\left\{\dfrac{1}{(s-2)^2}\right\}$

(B) $\mathcal{L}^{-1}\left\{\dfrac{3}{s-2}\right\} + \mathcal{L}^{-1}\left\{\dfrac{1}{(s-1)}\right\}$

(C) $\mathcal{L}^{-1}\left\{\dfrac{2}{s-2}\right\} + \mathcal{L}^{-1}\left\{\dfrac{2}{(s-1)^2}\right\}$

(D) $\mathcal{L}^{-1}\left\{\dfrac{2}{s-2}\right\} - \mathcal{L}^{-1}\left\{\dfrac{1}{(s-2)^2}\right\}$

(E) $\mathcal{L}^{-1}\left\{\dfrac{1}{s-2}\right\} - \mathcal{L}^{-1}\left\{\dfrac{3}{(s-2)^2}\right\}$

7. If you solve the following system of differential equations

$$\begin{cases} x' = x + 3y \\ y' = 5x + 3y \end{cases}$$

subject to the initial condition $x(0) = 5$ and $y(0) = 3$, then $x(t)$ is given by:

A $x = e^{2t} + 4e^{5t}$ B $x = 3e^{-2t} + 2e^{6t}$ C $x = 2e^{-2t} + 3e^{6t}$

D $x = 3e^{2t} + 2e^{5t}$ E $x = 6e^{2t} - e^{3t}$

8. Find c_4, where $y = \sum_{k=0}^{\infty} c_k x^k$ is a solution to the differential equation

$$y'' - (1 + x^2)y = 0\,,$$

subject to the initial condition $y(0) = 8$ and $y'(0) = -3$.

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

9. Find a recurrence relation for c_n, where $y = \sum_{k=0}^{\infty} c_k x^k$ is a solution to the differential equation

$$y'' + xy' + y = 0\,.$$

That is, give a formula for c_{n+2} in terms of c_n and/or c_{n+1}.

(A) $c_{n+2} = -\dfrac{c_n + c_{n+1}}{n}$

(B) $c_{n+2} = \dfrac{(2n-1)}{(n+1)(n+2)} c_n$

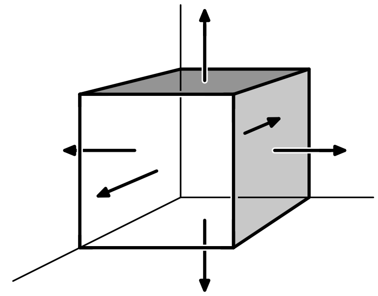
(C) $c_{n+2} = -\dfrac{c_n}{n+2}$

(D) $c_{n+2} = -\dfrac{c_n + c_{n+1}}{n(n+1)}$

(E) $c_{n+2} = -\dfrac{c_n}{2n(n-1)}$

10. Let $\mathcal{S}$ be the surface consisting of the boundary of the unit cube $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$, and let $\mathbf{F} = (e^x + z)\mathbf{i} + (y^2 - x)\mathbf{j} - xe^y\mathbf{k}$. Evaluate the outward flux (or divergence)

$$\oiint_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S} = \oiint_{\mathcal{S}} \mathbf{F} \cdot \mathbf{n}\, dS\,,$$



also written

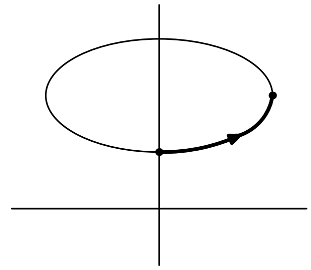
$$\oiint_{\mathcal{S}} (e^x + z)\, dy\, dz + (y^2 - x)\, dz\, dx - xe^y\, dx\, dy\,.$$

(A) $e - 2$ (B) e (C) $e + 2$ (D) $e + 3$ (E) $4 - e$

11. Find

$$\int_{\mathcal{C}} (2x + e^y)\, dx + (3y^2 + xe^y)\, dy\,,$$

where $\mathcal{C}$ is the arc of the ellipse $x^2 + 4(y-2)^2 = 4$ from $(0,1)$ to $(2,2)$ in the counterclockwise direction.



(A) $e^3 + 12$ (B) $3e - 7$ (C) $2e^2 + 11$ (D) $3e^2 + 14$ (E) $e^2 + 8$

12. Evaluate

$$\oint_{\mathcal{C}} (e^x + 3y)\,dx + (4x + y^6)\,dy$$

where $\mathcal{C}$ is the circle $(x-2)^2 + (y-4)^2 = 1$, traversed counterclockwise.

(A) 0 (B) 1 (C) e (D) π (E) 4

13. A certain forced undamped oscillator is modelled by the differential equation

$$m\frac{d^2x}{dt^2} + 18x = 4\cos 3t\,.$$

What mass $m > 0$ corresponds to resonance, that is, $x(t)$ is unbounded as $t \to \infty$?

(A) $m = 1$ (B) $m = 2$ (C) $m = 3$ (D) $m = 4$ (E) $m = 5$

14. Compute the determinant

$$\begin{vmatrix} 0 & -1 & -3 \\ 2 & 3 & 3 \\ -2 & 1 & 1 \end{vmatrix}.$$

(A) -16 (B) -8 (C) 0 (D) 8 (E) 16

15. Solve for x in the system

$$\begin{aligned} x - y - 3z &= 0 \\ x + 3y + 3z &= 2 \\ y + z &= -2 \end{aligned}$$

As a hint:

$$\begin{vmatrix} 1 & -1 & -3 \\ 1 & 3 & 3 \\ 0 & 1 & 1 \end{vmatrix} = -2$$

You can also use the previous problem.

(A) -8 (B) -4 (C) no solution (D) 4 (E) 8

16. Compute the sum of the elements in the last column of A^{-1} where

$$A = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}.$$

(A) -1 (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) 1 (E) A is actually singular, and A^{-1} is undefined.

17. If

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = x\,,$$

then

$$\begin{vmatrix} 2c_2 + a_2 & b_2 & -a_2 \\ 2c_1 + a_1 & b_1 & -a_1 \\ 2c_3 + a_3 & b_3 & -a_3 \end{vmatrix} = ?$$

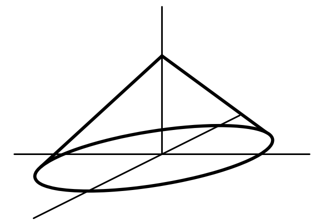
(A) 0 (B) $-x$ (C) $2x$ (D) $-2x$ (E) $3x$

18. Select a matrix with eigenvalues 0 and 2, and corresponding eigenvectors $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

(A) $\begin{pmatrix} -4 & 2 \\ -12 & 6 \end{pmatrix}$ (B) $\begin{pmatrix} -2 & 1 \\ -8 & 4 \end{pmatrix}$ (C) $\begin{pmatrix} -4 & -12 \\ 2 & 6 \end{pmatrix}$ (D) $\begin{pmatrix} 0 & 3 \\ 0 & 2 \end{pmatrix}$ (E) $\begin{pmatrix} -4 & -8 \\ 3 & 6 \end{pmatrix}$

19. The characteristic polynomial of $A = \begin{pmatrix} 8 & -2 & 2 \\ -2 & 5 & 4 \\ 2 & 4 & 5 \end{pmatrix}$ is $p(\lambda) = -\lambda(\lambda - 9)^2$. $\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ is an eigenvector for 0, and $\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$ is an eigenvector for 9. Select the vector below which is another eigenvector for 9, and is also orthogonal to $\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$.

(A) $\begin{pmatrix} 0 \\ 0 \\ 9 \end{pmatrix}$ (B) $\begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix}$ (C) $\begin{pmatrix} 4 \\ 8 \\ 2 \end{pmatrix}$ (D) $\begin{pmatrix} 3 \\ 6 \\ -1 \end{pmatrix}$ (E) $\begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix}$



20. Let $\mathbf{F} = (x-y)\mathbf{i} - 2xz\mathbf{j} - x^2\mathbf{k}$. Evaluate

$$\iint_{\mathcal{S}} (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS = \iint_{\mathcal{S}} (\nabla \times \mathbf{F}) \cdot d\mathbf{S},$$

where $\mathcal{S}$ is the portion of the cone $z = 1 - \sqrt{x^2+y^2}$ above the xy-plane with outward directed normal (away from the origin).

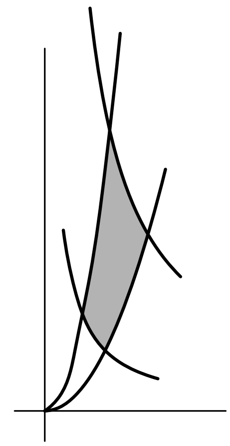
Equivalently, evaluate $\iint_{\mathcal{S}} P\, dy\, dz + Q\, dz\, dx + R\, dx\, dy$ where $\nabla \times \mathbf{F} = P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}$.

(A) 0 (B) π (C) 2π (D) 3π (E) 4π

21. Evaluate the integral

$$\iint_{\mathcal{R}} xy\,dA$$

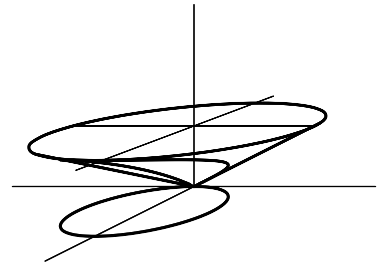
over the region bounded by the curves $xy = 1$, $xy = 5$, $y = x^2$ and $y = 4x^2$.



(A) $2\ln 2$ (B) $4\ln 2$ (C) $8\ln 2$

(D) $12\ln 2$ (E) $15\ln 2$

22. Find the surface area of the portion of the cone $z = \frac{1}{2}\sqrt{x^2 + y^2}$ lying above the circle $(x-1)^2 + y^2 = 1$.



(A) $\dfrac{4}{3}\pi$ (B) $\pi \ln 3$ (C) $\dfrac{\sqrt{5}}{2}\pi$

(D) $2\pi - 3$ (E) $\pi + 1$