Math 240 **FINAL EXAM** **May 4, 2010**

Circle one: **Professor Ziller**

Professor Zywina

Name: ______________________________

Penn Id#: ____________________

Signature: ______________________________

TA: ______________________________

Recitation Day and Time: ____________________

You need to show your work, even for multiple choice problems. A correct answer with no work will get 0 points. The only exception are True/False problems, where no work needs to be shown. Each problem is worth 10 points.

You are NOT allowed to use a calculator. The extra double sided sheet of paper needs to be hand written in your own hand writing (no copies allowed).

(Do not fill these in; they are for grading purposes only.)

1)	9)
2)	10)
3)	11)
4)	12)
5)	13)
6)	14)
7)	15)
8)	

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Total

1. For what values of a does the following system have infinitely many solutions?

$$\begin{aligned} x + y + z &= 1 \\ 2x + ay + z &= 0 \\ -x + y + z &= 3 \end{aligned}$$

Answer:

(a) 0 (b) 3 (c) 2 (d) 1 (e) 4 (f) -8

2. (a) Find the rank of the matrix

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

(b) What is $\det A$?

(c) The set of solutions of the homogeneous system $AX = 0$ depends on how many arbitrary parameters?

3. Let A be the matrix

$$A = \begin{pmatrix} -5 & -4 \\ 8 & k \end{pmatrix}$$

For which value of k does there exist an invertible matrix P such that

$$PAP^{-1} = \begin{pmatrix} -1 & 0 \\ 0 & 3 \end{pmatrix}$$

Answer:

(a) 2 (b) 7 (c) -4 (d) 8 (e) -2 (f) -8

4. Find the maximal number of linearly independent eigenvectors for the matrix

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 4 \\ 0 & -1 & 4 \end{pmatrix}$$

5. Suppose that a pair of solutions $(x(t), y(t))$ to the system of differential equations

$$\begin{aligned} x' &= -x + ky \\ y' &= \qquad\quad y \end{aligned}$$

satisfies $\lim_{t\to+\infty} \dfrac{x(t)}{y(t)} = 1$ where k is some unknown constant. What is k?

Answer:

(a) 2 (b) 4 (c) -4 (d) 8 (e) -2 (f) -8

6. Find the general solution to the system

$$X' = \begin{pmatrix} 2 & -4 \\ 0 & 2 \end{pmatrix} X$$

7. Let $y(t)$ be the solution of

$$y'' - 2y' + y = 2e^t$$

which satisfies the initial conditions $y(0) = 0$ and $y'(0) = 2$. Find $y(1)$.

Answer:

(a) e (b) $1/e$ (c) $3e$ (d) $3/e$ (e) e^2 (f) -8

8. Find the solution of the differential equation

$$y'' + 2y' + 5y = 0$$

subject to the initial conditions $y(0) = 2$ and $y'(0) = 2$.

9. Find the solution $y(x)$ of

$$x^2y'' + xy' - y = 0$$

for which $y(1) = 2$ and $y'(1) = 4$.

10. Find the recurrence relationship for a power series solution of the differential equation

$$(x^2 + 1)y'' + xy' - y = 0$$

at $x = 0$. Determine the first 4 non-zero terms of the series for the solution with $y(0) = 1,\ y'(0) = 2$.

11. Find the work done by the force field

$$\mathbf{F}(x, y, z) = e^y\,\mathbf{i} + (xe^y + e^z)\,\mathbf{j} + ye^z\,\mathbf{k}$$

in moving a particle from $(1, 0, 0)$ to $(0, 1, \pi)$ along the helix $x = \cos(t)$, $y = \sin(t)$, $z = t$.

Answer:

(a) e^π (b) $e^\pi - 2$ (c) $e^\pi + 1$ (d) $e^\pi - 1$ (e) $2e^\pi - 1$ (f) $2e^\pi - 3$

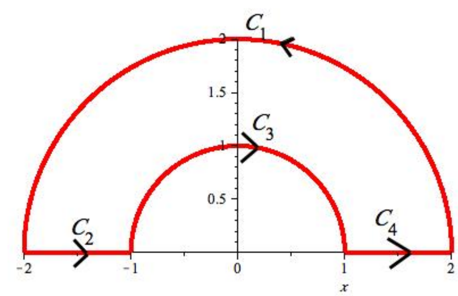
12. Let C be the curve that is the intersection of the plane $x+y+z=1$ and the cylinder $x^2+y^2=9$ oriented counter-clockwise as viewed from above. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$ where

$$\mathbf{F}(x,y,z) = -yx^2\,\mathbf{i} + y^2z\,\mathbf{j} + z^2\,\mathbf{k}.$$

Answer:

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) -2 (e) $-\frac{1}{4}$

13. Let $\mathbf{F}(x, y) = \langle y^2, 3xy \rangle$ be a vector field in the plane and let C be the closed curve shown in the following picture with a counter-clockwise orientation [the curve C_1 and C_3 travel along a circle of radius 2 and 1, respectively]. Evaluate the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$.



Answer:

(a) -10 (b) $\frac{14}{3}$ (c) $\frac{10}{3}$ (d) $-\frac{10}{5}$ (e) $\frac{3}{2}$

14. Find the outward flux $\iint_S \mathbf{F} \cdot \mathbf{n}\ \mathbf{dS}$ of the vector field $\mathbf{F} = 3xy^2\mathbf{i} + 3yz^2\mathbf{j} + 3zx^2\mathbf{k}$ where the surface S is the boundary of the region $1 \leq x^2 + y^2 + z^2 \leq 4$.

15. This is the only problem where no work needs to be shown. Each answer is worth one point.

Which one of the following statements are true or false:

(a) If $\det(A - I) = 0$, then 1 is an eigenvalue of A. True◯ False◯

(b) If $\det A = 0$, then the system $AX = 0$ has a unique solution. True◯ False◯

(c) $(AB)^T = A^T B^T$ for all square matrices A, B. True◯ False◯

(d) If S is any closed surface, then $\iint_S \text{curl}\mathbf{F} \cdot \mathbf{n} dS = 0$. True◯ False◯

(e) If $\mathbf{F} = \nabla f$ then $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ for all closed curves C. True◯ False◯