Math 240, MAKEUP FINAL EXAM

January 16, 2013

INSTRUCTIONS	OFFICIAL USE ONLY

INSTRUCTIONS:

1. Please complete the information requested below. There are 8 multiple choice problems, 1 True/False problem, and 1 open answer problem. Partial credit will be given on the multiple choice questions.

2. Please show all your work on the exam itself. Correct answers with little or no supporting work will not be given credit.

4. You are allowed to use one hand-written sheet of paper with formulas. *No calculators, books or other aids are allowed.* Please turn in your crib sheet together with your exam.

• **Name** (please print):

• **Name of your Professor:**
◯ **Ryan Blair** ◯ **Vasile Brinzanescu**
◯ **Tony Pantev**

• **Name of your TA:**
◯ **Shiying Dong** ◯ **Ryan Eberhart**
◯ **Ryan Manion** ◯ **Sebastian Moore**

• **Recitation day and time:**

• **My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this exam.**
Signature:

Problem	Points
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
Total	

(1) $\boxed{\text{10 POINTS}}$ Which of the following statements are **false**?

(i) A parametrized surface S is always a graph of a function.

(ii) The surface $\overrightarrow{X}(s,t) = \langle s^2 + t^2, -2s\cos(t), -2s\sin(t)\rangle$, $0 < s < 1$, $0 < t < 2$ is orientable.

(iii) Let S be the unit sphere oriented with the outward normal vector. Let $\overrightarrow{F}$ be a vector field such that $\operatorname{div}\overrightarrow{F} = 6$ everywhere in $\mathbb{R}^3$. Then the flux of $\overrightarrow{F}$ through S is 12π.

(a) (i) only

(b) (ii) only

(c) (iii) only

(d) (i) and (ii) only

(e) (i) and (iii) only

(f) (ii) and (iii) only

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(2) $\boxed{\text{10 POINTS}}$ Let S be the part of the graph of the function $f(x,y) = 3 - x^2 - y^2$ that lies over the disk $x^2 + y^2 \leq 1$ in the xy-plane. Suppose S is oriented by the upward pointing normal vector. Compute the flux $\iint_S \overrightarrow{F} \cdot d\overrightarrow{S}$ of the vector field

$$\overrightarrow{F} = \left\langle 5x + \frac{x^2+y^2}{3+2y^2},\ 2y + \sin(x),\ 11 - 7z - \frac{2xz}{3+2y^2} \right\rangle.$$

Hint: Use Gauss's theorem.

(a) 11π

(b) 24

(c) 0

(d) 7π

(e) 2π

(f) $5+\pi$

(3) $\boxed{\text{10 POINTS}}$ True or false. To receive *any* credit, you must also give a reason or a counterexample for each statement.

(a) If A and B are 4×4 matrices such that $\operatorname{rank}(AB) = 3$, then $\operatorname{rank}(BA) < 4$.

(b) If A is a 5×3 matrix with $\operatorname{rank}(A) = 2$, then $Ax = b$ will have a solution for every vector b in $\mathbb{R}^5$.

(c) If A is a 4×7 matrix, then A^T and A have the same rank.

(d) If A and $B \neq 0$ are 2×2 matrices such that $AB = BA = 0$, then A must be the zero matrix.

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(4) $\boxed{\text{10 POINTS}}$ Please circle "**T**" for true or "**F**" for false in the space provided to the left of the following statements. You **do not** have to justify your answer to receive full credit.

T **F** The vector space of all 4×4 matrices that are both symmetric and skew-symmetric has dimension one.

T **F** If $T : V \to W$ is a linear transformation between vectorspaces V and W, then the set $\{v \in V | T(v) = \mathbf{0}\}$ is a vector subspace of V.

T **F** The vectors $v_1, v_2, \ldots, v_n$ in $\mathbb{R}^n$ are linearly independent if and only if $\text{span}\{v_1, v_2, \ldots, v_n\} = \mathbb{R}^n$.

T **F** If A is an $n \times n$ matrix such that $nullity(A) = 0$ then A is the identity matrix.

T **F** If A is an $m \times n$ matrix with rank m, then the columns of A are linearly independent.

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(5) $\boxed{\text{10 POINTS}}$ Which o fthe following collections of matrices form a subspace of the vector space $\mathrm{Mat}_{2\times 2}(\mathbb{R})$?

(i) All non-singular matrices: $\det A \neq 0$;

(ii) All nilpotent matrices: $A^2 = 0$;

(iii) All matrices A whose transpose commutes with a fixed matrix B: $A^T B = BA^T$;

(iv) All skew-symmetric matrices: $A^T = -A$.

(a) only (i)

(b) only (ii)

(c) only (iii)

(d) only (iv)

(e) only (i) and (ii)

(f) only (iii) and (iv)

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(6) $\boxed{\text{10 POINTS}}$ Produce a matrix A with the following properties

1. A has eigenvalues -1 and 2;

2. the eigenvalue -1 has eigenvector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$;

3. the eigenvalue 2 has eigenvectors $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$.

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(7) $\boxed{\text{10 POINTS}}$ Let A be an $n \times n$ matrix with real entries, and let λ be an eigenvalue of A. Which of the following statements is **correct**?

(i) $\alpha\lambda$ is an eigenvalue of αA for all real scalars α;

(ii) λ^2 is an eigenvalue of A^2;

(iii) $\lambda^2 + \alpha\lambda + \beta$ is an eigenvalue of $A^2 + \alpha A + \beta I_n$ for all real scalars α and β;

(iv) If $\lambda = a + ib$ with $a, b \neq 0$ are some real numbers, then $\bar{\lambda} = a - ib$ is also an eigenvalue of A.

(a) only (i)

(b) only (ii)

(c) only (iii)

(d) only (iv)

(e) (i), (ii), (iii), and (iv)

(f) only (ii) and (iv)

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(8) $\boxed{\text{10 POINTS}}$ Let $\mathbf{x}' = A\mathbf{x}$ be a vector differential equation, where A is a 2×2 matrix with real entries. Suppose we know that one solution to this equation is given by $e^t \begin{bmatrix} \sin(2t) \\ \cos(2t) \end{bmatrix}$. Find the matrix A and the solution to $\mathbf{x}' = A\mathbf{x}$ that satisfies the initial condition $\mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

(a) $A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$, $\quad \mathbf{x}(t) = e^t \begin{bmatrix} \cos(2t) \\ -\sin(2t) \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $\quad \mathbf{x}(t) = e^t \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$, $\quad \mathbf{x}(t) = e^t \begin{bmatrix} \sin(2t) \\ \cos(2t) \end{bmatrix}$

(d) $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\quad \mathbf{x}(t) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

(e) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\quad \mathbf{x}(t) = e^t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(f) $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $\quad \mathbf{x}(t) = e^t \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(9) $\boxed{\text{10 POINTS}}$ Which of the following functions is a particular solution to the nonhomogeneous linear differential equation

$$y'' - 2y' + y = xe^x$$

(a) e^x

(b) $\frac{1}{6}e^x$

(c) xe^x

(d) $\frac{1}{3}xe^x$

(e) $\frac{1}{6}x^3e^x$

(f) $\frac{1}{2}x^2e^x$

(10) $\boxed{\text{10 POINTS}}$

Let $\mathbf{x} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ be the vector valued function that solves the initial value problem

$$\mathbf{x}' = \begin{bmatrix} -1 & 0 \\ 4 & -1 \end{bmatrix} \mathbf{x}, \qquad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

What is $x_2(2)$?

(a) e^{-2}

(b) 0

(c) $8e^{-2}$

(d) 5

(e) $5e^{-1}$

(f) $2e^{-1}$
