MATHEMATICS DEPARTMENT
UNIVERSITY *of* PENNSYLVANIA

Mathematics 240 Your Name: ______________________________

Final Examination Fall 2005

Check one: ❑Prof. Crotty ❑Prof. Schneiderman ❑Prof. Vogel

TA's Name: ____

You have two hours for this examination. Show your work in the space provided. Write your answers in the appropriate spaces below. Write clearly—if your answer cannot be read, it is *wrong*. No part credit is given in the multiple choice part of this exam. Part credit *may* be given in the free response part, so be sure to show as much detail as needed to determine how you reached your answer.

Multiple Choice:

Write the letter of your answer in the space next to the question number. Questions with ambiguous or multiple answers will be marked wrong.

1		6		11	
2		7		12	
3		8		13	
4		9		14	
5		10a		15a	
		10b		15b	
		10c		15c	
		10d		15d	

Free Response questions:

Record your answer in the space provided. Show your work in the space provided in the Exam booklet.
Partial credit will only be given if the work shown is clearly appropriate and, properly executed, would lead to a correct solution of the problem.

1.	3.
2.	4.

Multiple Choice Problems:

Do your work in the space provided; you may use the backs of the previous sheets if you need more room. An answer with no supporting work may receive NO CREDIT even if correct. Problems in this part of the exam will be marked as correct or incorrect; NO PARTIAL CREDIT will be given, so work quickly but carefully. Record your answers on your answer sheet. Each question is worth 4 points.

1. For what value of the constant b is the vector field $\mathbf{F} = bxy\mathbf{i} + x^2y\mathbf{j}$ conservative?

a) 0 b) ¼ c) ½ d) ¾ e) 1 f) no such b exists

2. Evaluate $\int_C \mathbf{F} \bullet d\mathbf{r}$ if $\mathbf{F} = 2xy^3z^4\mathbf{i} + 3x^2y^2z^4\mathbf{j} + 4x^2y^3z^3\mathbf{k}$ and C: $x = t$, $y = t^2$, $z = t^3$, $0 \le t \le 2$.

a) 0 b) 2 c) 4 d) 8 e) 16 f) 2^{20}

3. Evaluate $\int_C x\,dy$ along the triangular path from (0, 0) to (2, 3) to (1, 0) to (0, 0).

a) ½ b) 1 c) 3/2 d) 2 e) –3/2 f) –1/2

4. Evaluate the surface integral $\iint_S z^2 dS$ where S is the part of the cylinder $z = \sqrt{1 - x^2}$ that lies above the square with vertices (–1, –1), (1, –1), (–1, 1) and (1, 1).

a) π b) $\sqrt{2}$ c) 2π d) 2 e) $\sqrt{8}$ f) 4

5. Let $\mathbf{F}(x,y,z) = xy\mathbf{i}$ and let S be the surface of the solid $E = \{(x,y,z) | 0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le z \le 1\}$.

Evaluate $\iint_E \mathbf{F} \bullet d\mathbf{S}$.

a) 0 b) ¼ c) ½ d) ¾ e) 1 f) 5/4

6. In solving the differential equation $y' + x^4 y = 0$ by use of the power series $\sum_{n=0}^{\infty} a_n x^n$, what is the first value of n beyond $n = 0$ for which the coefficient a_n can be non-zero?

a) 1 b) 2 c) 3 d) 4 e) 5 f) 6

7. The first three non-zero term in the power series solution of $y'' - xy' - y = 0$ subject to $y(0) = 0,\ y'(0) = 1$ are:

a) $x - x^2 + x^3$ b) $x + x^2/2 + x^3/3$ c) $x + x^2/2 + x^3/3$

d) $x + x^3/3 + x^5/5$ e) $x + x^3/6 + x^5/120$ f) $x - x^2/2 + x^3/6$

8. The location and there type of the singular points of the differential equation $y'' - xy' - y = 0$ are:

a) $x = 0$, regular; $x = 1$, regular; $x = -1$, regular b) $x = 0$, irregular; $x = 1$, regular; $x = -1$, regular

c) $x = 0$, regular; $x = 1$, irregular; $x = -1$, regular d) $x = 0$, regular; $x = 1$, regular; $x = -1$, irregular

e) $x = 0$, regular; $x = 1$, irregular; $x = -1$, irregular f) $x = 0$, irregular; $x = 1$, irregular; $x = -1$, irregular

9. Consider the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 4 & 4 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. If this is the augmented matrix [A | **b**] of the linear system $A\mathbf{x} = \mathbf{b}$ after Gaussian reduction has been performed on it, then the system is:

a) consistent with a unique solution
b) consistent with exactly two solutions
c) consistent with infinitely many solutions
d) inconsistent
e) can't tell from the matrix as shown

10. Mark each statement as *True* or *False*. Write the **word** True or False in the appropriate space on your answer sheet.

a) The column vectors of any 4x5 matrix must be linearly dependent.
b) Assume that the vectors **x** and **y** are linearly independent. If the set {**x**, **y**, **z**} is linearly dependent, then **z** can be expressed as $c_1\mathbf{x} + c_2\mathbf{y}$ where c_1 and c_2 are some constants.
c) Suppose the eigenvalues of a matrix are 2, 2, and 4 and the corresponding eigenvectors are $\mathbf{v}_1 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ corresponding to $\lambda = 2$ and $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ corresponding to $\lambda = 4$, Then A can be diagonalized using the given eigenvalues and eigenvectors.
d) If $\det(A) = 5$ and $\det(B) = 10$ then $\det(A + B) = 15$.

11. Let $\mathbf{F}(x, y, z) = z^2\mathbf{i} + x^2\mathbf{j} + y^2\mathbf{k}$. Find curl($\mathbf{F}$).

a) $2y\mathbf{i}$ **b)** $2z\mathbf{j}$ c) $2x\mathbf{k}$ d) $2y\mathbf{i} + 2z\mathbf{j} + 2x\mathbf{k}$ e) $2y\mathbf{i} - 2z\mathbf{j} + 2x\mathbf{k}$

12. Evaluate $\int_C \mathbf{F} \bullet d\mathbf{r}$ where $\mathbf{F}(x, y) = 2xy\mathbf{i} + x^2y\mathbf{j}$ and C is given by $\mathbf{r}(t) = t^2\mathbf{i} + t^3\mathbf{j}$, $0 \le t \le 2$.

a) 127 b) 128 c) 63 d) 31 e) 16 f) 15

13. Solve $x^2y'' + 5xy' + 4y = 0$

a) $y = (c_1 + c_2\ln x)x^{-2}$ b) $y = c_1x^{-2} + c_2x^{-1}$ c) $y = c_1x^{-2} + c_2x^2$

d) $y = c_1x^{-2} + c_2\ln x$ e) $y = c_1 x^{-2} + c_2x^{-2}$ f) $y = c_1\cos(-2\ln x) + c_2\sin(-2\ln x)$

14. If the differential equation $(3x - 2)y'' + 6xy' - y = 0$ is solved by a power series expansion about $x = 0$, the resulting power series will have a radius of convergence of at least:

a) 1 b) 2/3 c) 3/2 d) 2 e) 3 f) ∞

15. Mark each statement as *True* or *False*. Write the **word** True or False in the appropriate space on your answer sheet.

a) Any set of five vectors in $\Re^5$ is a basis for $\Re^5$.

b) The matrix $\begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ has two linearly independent eigenvectors corresponding to the same eigenvalue.

c) For any real constants a, b and c, $\mathbf{F} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ is a constant vector field. If S is the sphere of radius$r$ centered at the origin, then the flux of **F**, $\iint_S (\mathbf{F} \bullet \mathbf{n})dS$, out of S is zero.

d) If y_1 and y_2 are solutions of the differential equation ,$(3x-2)y'' - 2y\cos x = 0$ then $5y_1 + y_2$ is also a solution.

Free Response Questions:
Give a solution for each problem in the space provided. Be clear and complete. Place your answer in the space provided on the Answer Sheet and in the location provided with the problem. Part credit will be awarded as appropriate. Each problem is worth 10 points.

1. Solve the linear system given by the method of your choice:

$$\begin{aligned} 2x &+ y &- 2z &= 10 \\ 3x &+ 2y &+ 2z &= 1 \\ 5x &+ 4y &+ 3z &= 4 \end{aligned}$$

$x =$ __________, $y =$ __________, $z=$ __________

2. Solve the system of linear first order ODEs:

$$\mathbf{Y}' = \begin{bmatrix} -2 & -2 \\ -1 & -3 \end{bmatrix}\mathbf{Y} \text{ where } \mathbf{Y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$y_1 =$ ____________________

$y_2 =$ ____________________

3. Compute: $\int_C 2y\,dx + 5x\,dy$ where $\boldsymbol{C}$ is the circle $(x-1)^2 + (y+3)^2 = 25$ with positive orientation.

$\int_C 2y\,dx + 5x\,dy =$ ______________________

4. Solve the differential equation $y'' + 2xy' - 4y = 0$ by a power series expansion about $x = 0$. You are done when you have derived the recurrence relation for the coefficients and written out the first three terms of each series in the solution.

coefficient recurrence: ______________________________

y_1 = ______________________________________

y_2 = ______________________________________