UNIVERSITY *of* PENNSYLVANIA
MATHEMATICS DEPARTMENT
Mathematics 240 – Final Exam
Fall 2004

***Your Name:*_________________________** **Penn *ID#*______________________**

Professor (Check one): Crotty ❑ 001 ❑ 601 Preston ❑ 002 Gerstenhaber ❑ 003

TA's name:____

There are 12 questions on this exam. You are required to do all of them.

You have 2 hours to complete this examination.

You are permitted the use of a one page (8.5" x 11") notes page. Both sides may be used.

NO calculators, cell phones, iPods, etc. are permitted.

Please show all work in the space provided on your test paper and write your answers in the appropriate place on each page. If you need more space, use the back of the page facing the problem on which you are working.

DO NOT DETACH THIS SHEET FROM YOUR TEST

---*Please do not write below this line*---

Scores:

1. (10) __________	**2. (10)** __________	**3. (10)** __________	**4. (10)** __________
5. (10) __________	**6. (10)** __________	**7. (10)** __________	**8. (10)** __________
9. (10) __________	**10. (10)**__________	**11. (10)**__________	**12.(10)**__________

Total __________ **Scaled** __________

Name: ____

1. Each of the matrices below is a row echelon form of the augmented matrix of a system of linear equations. Determine if each system is consistent or inconsistent and briefly justify your answer. If the system is consistent, find the solution.

a) $\begin{bmatrix} 1 & 3 & 4 & 6 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$	**Check One:** consistent ❑ inconsistent ❑	**Solution (if one exists):**

b) $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 5 \\ 0 & 1 & 3 & 3 & 5 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	**Check One:** consistent ❑ inconsistent ❑	**Solution (if one exists):**

c) $\begin{bmatrix} 1 & 0 & 2 & 0 & 4 \\ 0 & 0 & 1 & 2 & 6 \\ 0 & 0 & 1 & -5 & 0 \\ 0 & 0 & 0 & 1 & 6 \end{bmatrix}$	**Check One:** consistent ❑ inconsistent ❑	**Solution (if one exists):**

Name: ____

2. Solve $\mathbf{X}' = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix} \mathbf{X} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t}$ subject to $\mathbf{X}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Name: ____

3. a) Show that the vectors $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ *and* $\mathbf{u}_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ are linearly independent.

b) Express the vector $\mathbf{v} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ in the basis $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$.

v = ________ $\mathbf{u}_1$ + _________ $\mathbf{u}_2$ + _________ $\mathbf{u}_3$

Name: ____

4. a) Use Laplace Transforms (see attached list of transforms) to solve $y'' - 4y' = 6e^{3t} - 3e^{-t}$, $y(0) = 1$, $y'(0) = -1$.

Name: ____

5. Use diagonalization to solve the system $\mathbf{X}' = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\mathbf{X}$

Name: ____

6. Decide if $x = 0$ is an ordinary point, a regular or an irregular singularity for the following ODE. $(e^x - 1 - x)y'' + xy = 0$. Briefly justify your decision *DO NOT SOLVE THE EQUATION!*

   Hint: the Maclaurin series for e^x is $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$

Name: ____

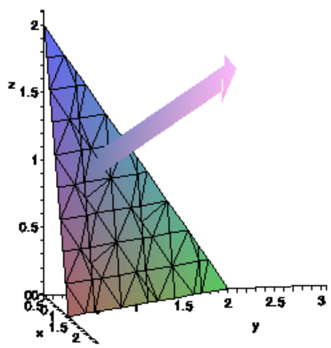
7. Use an appropriate power series method about $x = 0$ to find the two solutions of the given ODE.
$2xy'' + y' + y = 0$

Name: ____

8. Use Green's Theorem to evaluate $\oint_C y^2 dx + x^2 dy$ where C is the triangle bounded by $x=0$, $x + y = 1$, and $y = 0$.

Name: ____

9. Use Stokes Theorem to evaluate $\oint_C \mathbf{F} \bullet d\mathbf{r}$ if $\boldsymbol{F}=xz\mathbf{i} + xy\mathbf{j} + 3xz\mathbf{k}$ and C is the boundary of the portion of the plane $2x + y + z = 2$ in the first octant traversed counterclockwise as viewed from above (see diagram; the arrow is a normal vector to the plane).



Name: ____

10. Show that the given integral is path independent, then evaluate the integral two ways: a) find a function $d\phi = Pdx + Qdy$ and, b) integrate along *any* convenient path between the given points.

$$\int_{(0,0)}^{(\pi/2,0)} \cos x \cos y dx + (1 - \sin x \sin y) dy$$

Name: ____

11. Solve $x^2 y'' - 7xy' + 41y = 0$.

Name: ____

12. Evaluate the surface integral $\iint_S G(x,y,z)dS$ if $G(x,y,z) = x + y + z$ and S is the cone $z = \sqrt{x^2 + y^2}$ between z=1 and z=4 (see diagram).