Math 115 Makeup Final Exam Fall 2006

1. The tangent plane to the ellipsoid $\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3$ at the point (-2, 1,-3) intersects the x-axis at the point:

$$A.(4, 0, 0) \quad B.(3, 0, 0) \quad C.(-3, 0, 0) \quad D.(2, 0, 0)$$
$$E.(6, 0, 0) \quad F.(-6, 0, 0) \quad G.(1, 0, 0) \quad H.(-1, 0, 0)$$

2. Find the point(s) closest to (1,0) on the ellipse $\frac{x^2}{3} + y^2 = 1$.

$$A.\ (1, 0) \quad B.\ (0, \pm 1) \quad C.\ (\sqrt{3}, 0) \quad D.\ (\pm 3/2, 1/4) \quad E.\ (3/2, \pm 1/2)$$

3. Consider the function $f(x, y) = x^2 - 4x + y^3 - 3y$. Find its critical points and determine their type.

A. Local min at (2, 1), local min at (2, -1).
B. Local min at (2, 1), saddle at (2, -1).
C. Local min at (2, 1), local max at (2, -1).
D. Saddle at (2, 1), local min at (2,-1).
E. Saddle at (2,1), saddle at (2,-1).
F. Saddle at (2,1), local max at (2, -1).
G. Local max at (2, 1), local min at (2, -1).
H.Local max at (2, 1), Saddle at (2, -1).

4. If $z = e^x \sin(y)$, where $x = st^2$ and $y = s^2 t$, then $\frac{\partial z}{\partial s}$ when $s = 1, t = 1$ is:

$$A.\ 0 \quad B.\ \sin(1) \quad C.\ \cos(1) \quad D.\ e\sin(1) \quad E.\ e\cos(1)$$
$$F.\ 2e\cos(1) \quad G.\ e\sin(1) + 2e\cos(1) \quad H.\ e\sin(1) - 2e\cos(1).$$

5. Evaluate

$$\int_0^2 \int_y^2 e^{-\frac{x^2}{2}} dx dy$$

$$A.\ \frac{e^2 - 1}{8} \quad B.\ 2\ln(2) - 1 \quad C.\ \frac{1 - e}{2} \quad D.\ 1 - e^{-2}$$
$$E.\ \frac{e^2 - 1}{4} \quad F.\ 0 \quad G.\ e\sqrt{2} - 1. \quad H.\ \ln(2)$$

6. A bridge hand consists of 13 card from a standard 52-card deck. Find the probability that a bridge hand contains all four aces?

$$A.\ \frac{1}{13} \quad B.\ \frac{4}{13} \quad C.\ \frac{48!}{9!}\frac{13!}{52!} \quad D.\ 0$$
$$E.\ \frac{4!}{52!} \quad F.\ 1 - \frac{13!}{52!} \quad G.\ \frac{3}{4} \quad H.\ \frac{48!}{52!}$$

7. A jar contains 6 red balls and 4 green balls. If three balls are selected at random (without replacement) what is probability that there are more red balls than green balls?

$A.\ 3/10 \quad B.\ 1/2 \quad C.\ 3/5 \quad D. 6/11 \quad E.\ 2/3 \quad F.\ 7/11 \quad G.\ 83/120 \quad H.\ 7/13$

8. On a day when Tom operates the machinery, 70% of its output is high quality. On a day when Sally operates the machinery, 90% of its output is high quality. Tom operates the machinery 3 days out of 5. Three pieces of a random day's output were selected at random and 2 of them were found to be of high quality. What is the probability that Tom operated the machinery that day?

$$A.\ \frac{3}{5} \quad B.\ \frac{18}{32} \quad C.\ \frac{27}{49} \quad D.\ \frac{36}{125} \quad E.\ \frac{49}{67} \quad F.\ \frac{61}{84} \quad G.\ \frac{105}{188} \quad H.\ \frac{125}{201}$$

9. Suppose that a random variable X is uniformly distributed on the interval $[1, 6]$. The expected value of $1/X$ is:

$A.\ \ln(6) \quad B.\ 1/5 \quad C.\ \ln(6)/5 \quad D.\ \ln(6)/6 \quad E.\ \ln(5)/6 \quad F.\ 1/6 \quad G.\ 1/\ln(6) \quad H.\ 0$

10. The waiting time for an elevator is an exponentially distributed random variable with mean 3 minutes, i.e., it has probability density function $f(x) = \frac{1}{3}e^{-\frac{x}{3}}$ for $x \geq 0$. When your guest arrives at your floor you ask if he had to wait longer than 6 minutes for the elevator. He says no. What is the probability that he had to wait at least 3 minutes?

$$A.\ 1/2 \quad B.\ (e-1)/(e-2) \quad C.\ e^{-1}/e^{-2} \quad D.\ e^{-2}/e^{-1} \quad E.\ e^{-2}$$
$$F.\ e^{-1} - e^{-2} \quad G.\ (e^{-1} - e^{-2})/(e^{-2} - 1) \quad H.\ (e^{-1} - e^{-2})/(1 - e^{-2})$$

11. A fair coin is tossed 8 times. What is the probability that the number of heads in the first four tosses equals the number of heads in the last fours tosses? (e.g. HTTHHHTT they are equal, HHTHTTHH they are not equal. Note that $2^4 = 16,\ 2^5 = 32,\ 2^6 = 64,\ 2^7 = 128,\ 2^8 = 256$)

$$A.\ \frac{92}{256} \quad B.\ \frac{70}{256} \quad C.\ \frac{54}{256} \quad D.\ \frac{50}{256} \quad E.\ \frac{42}{256} \quad F.\ \frac{36}{256} \quad G.\ \frac{32}{256} \quad H.\ \frac{28}{256}$$

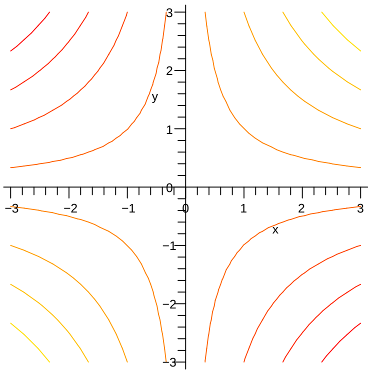
12. The number of transmission errors in any given time interval is a Poisson process. the average number of transmission errors is one error every two seconds. What is the probability there are three or more transmission errors in a four second interval.

$A.\ e^{-2} \quad B. 1-2e^{-2} \quad C. 1-5e^{-2} \quad D. 1-2/e \quad E.\ 1/e \quad F. 1-4e^{-2} \quad G. 5/e \quad H. 2/3$

13. Suppose that a particle starts at the origin of the real line and move along the line in jumps of one unit. For each jump, the probability is p $(0 \leq p \leq 1)$ that the particle will jump one unit to the left and the probability is $1-p$ that the particle will jumps one unit to the right. Find the expected value of positions of the particle after n jumps

$A.np(1-p)$ $B.n(1-2p)$ $C.n(2p-1)$ $D.np(2p-1)$ $E.np(1-3p)$ $F.n\dfrac{p}{1-p}$

14. The following is a contour plot of $z = f(x, y)$.



What is $f(x, y)$?

$$A.\ x^2 + y^2 \quad B.\ 2x^2 + y^2 \quad C.\ x^2 - 10y^2 \quad D.\ x^2 - y^2$$

$$E.\ x^2 y \quad F.\ x^2 - y \quad G.\ xy \quad H.\ x^2 + y$$

15. Which of the following statements regarding the system of equations

$$\begin{cases} x - y + z = 1 \\ x + 2y + 3z = 4 \\ 2x + 4y + 6z = k \end{cases}$$

is true ?

- A The system has a unique solution for any value of k.
- B The system has a unique solution only when $k = 8$.
- C The system has a unique solution only when $k = 0$.
- D The system only has infinitely many solutions when $k = 8$.
- E The system infinitely many solutions for every value of k.
- F The system has a unique solution for $k = 0$ and infinitely many when $k = 8$.
- G The system never has a solution.
- H The system has infinitely many solutions when $k > 0$, a unique solution when $k = 0$, and no solutions for $k < 0$.

16. Given that

$$A = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \text{ and } A \cdot B = \begin{pmatrix} 4 & 2 \\ 1 & -4 \end{pmatrix} \text{ find } B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

and compute the product $P = a + d$. Then $P =$

$$A.\ 2 \quad B.\ -2 \quad C.\ 1 \quad D.\ -1 \quad E.\ 0 \quad F.\ 3 \quad G.\ -3 \quad H.\ 4$$

17. A bank teller has a total of 70 bills in five-, ten-, and twenty-dollar denominations. The number of fives is three times the number of tens, while the total value of the money is \$960. Find the number (x_5, x_{10}, x_{25}) of each type of bill.

A. (20, 15, 18) B. (20, 8, 18) C. (24, 8, 17) D. (20, 50, 12)
E. (25, 15, 7) F. (10, 15, 18) G. (12, 8, 33) H. (24, 8, 38)

18. Three rods A, B and C are to be welded end to end to make 5 meter rod. The lengths of each of the rods is a normal distributed random variable with means μ and standard deviations σ given in the table below. What is the probability that the assembled rods will be within 1 millimeters of 5 meters? (i.e. $\Pr(|X_A + X_B + X_C - 5m| < 1cm)$ (1 centimeter $= 10^{-2}$meters)

A $\mu = 1$ meters $\sigma = 1$ centimeters
B $\mu = 2$ meters $\sigma = 2$ centimeters
C $\mu = 2$ meters $\sigma = 2$ centimeters

In the following answers, $\phi(x) = \int_0^x \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}\, dt$.

A. $\phi(\frac{1}{3})$ B. $\phi(\frac{1}{9})$ C. $2\phi(0.01)$ D. $2\phi(\frac{1}{3})$
E. $\phi(\frac{1}{5})$ F. $2\phi(0.02)$ G. $1 - \phi(\frac{1}{3})$ H. $1 - \phi(0.01)$.