Math 115 Final Exam Friday May 9, 2008 **Circle Instructor** Powers Storm

1. Consider the ellipsoid $\frac{x^2}{4}+\frac{y^2}{4}+2z^2=4$. Find the tangent plane at(x,y,z) = (2,2,1) and find where the plane intersects the z-axis.

z-intercept = A. 2 B. $\sqrt{2}$ C. $\frac{1}{\sqrt{2}}$ D. –1 E. 3

F. $-\sqrt{3}$ G. $\sqrt{5}$ H. 5

2. Find the minimum of the function f(x,y,z) = x + 4y + 9z on the surface $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=6$, with x > 0, y > 0, z > 0.

Minimum =

A. 1 B. $\sqrt{2}$ C. 2 D. 4 E. $\sqrt{18}$

F. 6 G. $2\sqrt{10}$ H. 14

3. The function $f(x,y) = -x^3 - 3xy + y^3$ has critical points at (0, 0) and (1, –1). Find their types:

A. $\begin{cases} \text{rel min at } x = 0, y = 0 \\ \text{rel min at } x = 1, y = -1 \end{cases}$ B. $\begin{cases} \text{rel min at } x = 0, y = 0 \\ \text{saddle at } x = 1, y = -1 \end{cases}$

C. $\begin{cases} \text{rel min at } x = 0, y = 0 \\ \text{rel max at } x = 1, y = -1 \end{cases}$ D. $\begin{cases} \text{rel max at } x = 0, y = 0 \\ \text{rel min at } x = 1, y = -1 \end{cases}$

E. $\begin{cases} \text{rel max at } x = 0, y = 0 \\ \text{saddle at } x = 1, y = -1 \end{cases}$ F. $\begin{cases} \text{saddle at } x = 0, y = 0 \\ \text{rel max at } x = 1, y = -1 \end{cases}$

G. $\begin{cases} \text{saddle at } x = 0, y = 0 \\ \text{rel min at } x = 1, y = -1 \end{cases}$ H. $\begin{cases} \text{saddle at } x = 0, y = 0 \\ \text{saddle at } x = 1, y = -1 \end{cases}$

4. Evaluate $\int_0^4 \int_{\frac{1}{2}y}^{2} e^{x^2}\, dx\, dy$

A. 1 B. e C. 2 D. $1 - e$ E. $e^4 - 1$ F. $\frac{1}{2}(e^2 - 1)$
G. $e^{1/2} - 1$ H. $e - 2$

5. Suppose the letters AABBC in a jar are drawn out one by one. What is the probability that the letters are drawn out in alphabetical order?

A. 1/120 B. 1/72 C. 1/42 D. 1/30 E. 2/61 F. 1/18
G. 1/9 H. 4/21

6. A pack of cards contains 12 cards numbered 1 to 12. Four cards are drawn without replacement. What is the probability that all the cards drawn are even numbers (i.e. four of the cards 2, 4, 6, 8, 10, 12). drawn out of the jar without replacement.

A. 5/64 B. 1/15 C. 1/33 D. 3/71 E. 1/9 F. 1/8
G. 3/16 H. 1/4

7. There are 5 red socks and 7 green socks in a drawer. Three socks are drawn out without replacement. What is the probablity there is a pair of red socks drawn (i.e., what is the probability there are two or more red socks.) Prob at least two red socks drawn =

A. 1/2 B. 1/3 C. 2/11 D. 3/11 E. 7/22 F. 4/11
G. 7/33 H.11/72

8. A fair coin is tossed until a heads occurs. Given the first heads occurs in the first three flips compute the expected number of flips. Expected number of flips =

A. 1 B. 5/4 C. 3/2 D. 11/7 E. 23/18 F. 2
G. 2½ H. 13/36

9. There are two coins A and B. Coin A is fair and B has a 1/3 probability of landing heads. Each coin is tossed twice. What is the probability that coin A produces more heads than B? Prob A > B =

A. 1/18 B. 1/9 C. 3/18 D. 4/9 E. 1/2 F. 5/9
G. 11/18 H. 2/3

10. There are 6 six sided dice. Die #1 has a spot on all six sides. Dice #2 and #3 have a spot on four sides and the remaining two sides blank. Dice #4, #5 and #6 have a spot on two sides and the remaining four sides are blank. One die is chosen at random and tossed twice and comes up with a spot showing both times. What is the probability it was the die #1 (with all a spot on all six sides)?

A. 1/3 B. 5/6 C. 11/18 D. 2/3 E. 5/8 F. 9/20
G. 3/4 H. 9/10

11. The joint probability distribution function for X and Y where $0 \le X \le 2$ and $0 \le Y \le 1$ is $f(x,y) = y$. Compute the probability that X > Y given X < 1. Hint Prob(X < 1) = 1/2 . Prob(X > Y | X < 1) =

A. 0 B. 2/9 C. 1/3 D. 4/9 E. 1/2 F. 5/9
G. 7/8 H. 15/16

12. The random variables X, Y, Z are uniformly distributed on the the cube $0 \le x \le 4$, $0 \le y \le 4$, $0 \le z \le 4$. Let T=X+Y+Z. Compute the variance of T. (Hint, X, Y and Z are independent). Var(T) =

A. 4 B. $2\sqrt{2}$ C. 7 D. 17/4 E. 5/3 F. 21/4
G. 3 H. 6

13. Suppose X and Y are independent exponentially distributed random variables both with mean two seconds. Compute the probability Pr(X > Y+1) i.e. that X occurs more than one second after Y. (The probability density function for an exponentially distributed random variable with mean *m* is $f(t) = \frac{1}{m} e^{-t/m}$, $t \ge 0$.

A. 1/3 B. 2/3 C. 1/2 D. e^{-2} E. $e^{-5/3}$ F. $1 - e^{-1}$
G. ½ $e^{-1/2}$ H. e/4

14. Two Geiger counters A and B are set to detect different radiation so they are statistically independent and the counting rate is a Poisson process with an average counting rate of 1 click per minute for A and 2 clicks per minute for B. What is the probablitity that the sum of the clicks from A and B is one or less in a given minute. (Hint A and B are independent. What is the probability that (A=0, B=0), (A=1, B=0), (A=0, B=1). Prob =

A. 1/3 B. $4e^{-3}$ C. $1-e^{-3}$ D. $5e^{-4}$ E. $4e^{-4}$ F. $\frac{3}{4}e^{-2}$

G. $7e^{-3}$ H. $(7/2)e^{-3}$

15. Given the equations $\begin{array}{rcrcrcl} 2x & + & y & + & 5z & = & 1 \\ -x & + & 3y & + & 4z & = & 1 \\ 3x & - & y & + & 3z & = & 0 \end{array}$. Find the value of x.

Note if $A = \begin{bmatrix} 2 & 1 & 5 \\ -1 & 3 & 4 \\ 3 & -1 & 3 \end{bmatrix}$ then $A^{-1} = \begin{bmatrix} 13 & -8 & -11 \\ 15 & -9 & -13 \\ -8 & 5 & 7 \end{bmatrix}$

A. -1 B. 0 C. 1 D. 2 E. 3 F. 4
G. 5 H. 6

16. For what values of k if any does the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & k \end{bmatrix}$ have an inverse?

A. all values of k. B. No values of k. C. Only for k = 6. D. Only for k≠ 6.
E. Only for k = 1. F. Only for k ≠ 1. G. Only for k = 2. H. Only for k≠ 2

17. Peter, Paul and Mary are playing catch. Peter and Paul throw to Mary 2/3 of the time and to each other 1/3 of the time. Mary throws to Paul and Peter with equal probability 1/2. What is the average probablility that Mary will have the ball in the long run?

A. 1/5 B. 1/6 C. 2/7 D. 3/7 E. 2/5 F. 2/11
G. 7/10 H. 4/11

18. A company is divided into three divisions, I, II and III. To produce a $1.00 worth of product in any division requires 10 cents spent in that division and 30 cents spent in the other two divisions. (e.g. To produce $1.00 of product in division I requires 10 cents in I, 30 cents in II and 30 cents in III). To meet a demand of 6 million of product I and no demand in II or III (i.e. these division simply support I) how should the production levels be set? Production level (1,11,111) in millions =

A. (5,3,3) B. (7,4,4) C. (8,3,3,) D.(9,4,4) E.(10,5,5) F.(11,6,6)
G.(12,5,5) H. (14,7,7)

19. Two large calculus classes take the final exam and the grades on the final are normally distributed. Class A has a mean score of 73 with a standard deviation of 15 and class B has as mean score of 68 with a standard deviation of 15. Suppose eight exams from class A are selected at random and eight exams from class B are selected at random. What is the probability that the average of the eight exams from class A is better than the average of the eight exams from class B. Use one of the following tables and indicate what you looked up and how you used it.

A. 95% B. 88% C. 75% D. 67% E. 58% F. 50%
G. 47% H. 25%

Table of the Standard Normal Distribution Function

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{(2\pi)^{1/2}} \exp\left(-\tfrac{1}{2}u^2\right) du$$

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0.00	0.5000	0.60	0.7257	1.20	0.8849	1.80	0.9641	2.40	0.9918
0.01	0.5040	0.61	0.7291	1.21	0.8869	1.81	0.9649	2.41	0.9920
0.02	0.5080	0.62	0.7324	1.22	0.8888	1.82	0.9656	2.42	0.9922
0.03	0.5120	0.63	0.7357	1.23	0.8907	1.83	0.9664	2.43	0.9925
0.04	0.5160	0.64	0.7389	1.24	0.8925	1.84	0.9671	2.44	0.9927
0.05	0.5199	0.65	0.7422	1.25	0.8944	1.85	0.9678	2.45	0.9929
0.06	0.5239	0.66	0.7454	1.26	0.8962	1.86	0.9686	2.46	0.9931
0.07	0.5279	0.67	0.7486	1.27	0.8980	1.87	0.9693	2.47	0.9932
0.08	0.5319	0.68	0.7517	1.28	0.8997	1.88	0.9699	2.48	0.9934
0.09	0.5359	0.69	0.7549	1.29	0.9015	1.89	0.9706	2.49	0.9936
0.10	0.5398	0.70	0.7580	1.30	0.9032	1.90	0.9713	2.50	0.9938
0.11	0.5438	0.71	0.7611	1.31	0.9049	1.91	0.9719	2.52	0.9941
0.12	0.5478	0.72	0.7642	1.32	0.9066	1.92	0.9726	2.54	0.9945
0.13	0.5517	0.73	0.7673	1.33	0.9082	1.93	0.9732	2.56	0.9948
0.14	0.5557	0.74	0.7704	1.34	0.9099	1.94	0.9738	2.58	0.9951
0.15	0.5596	0.75	0.7734	1.35	0.9115	1.95	0.9744	2.60	0.9953
0.16	0.5636	0.76	0.7764	1.36	0.9131	1.96	0.9750	2.62	0.9956
0.17	0.5675	0.77	0.7794	1.37	0.9147	1.97	0.9756	2.64	0.9959
0.18	0.5714	0.78	0.7823	1.38	0.9162	1.98	0.9761	2.66	0.9961
0.19	0.5753	0.79	0.7852	1.39	0.9177	1.99	0.9767	2.68	0.9963
0.20	0.5793	0.80	0.7881	1.40	0.9192	2.00	0.9773	2.70	0.9965
0.21	0.5832	0.81	0.7910	1.41	0.9207	2.01	0.9778	2.72	0.9967
0.22	0.5871	0.82	0.7939	1.42	0.9222	2.02	0.9783	2.74	0.9969
0.23	0.5910	0.83	0.7967	1.43	0.9236	2.03	0.9788	2.76	0.9971
0.24	0.5948	0.84	0.7995	1.44	0.9251	2.04	0.9793	2.78	0.9973
0.25	0.5987	0.85	0.8023	1.45	0.9265	2.05	0.9798	2.80	0.9974
0.26	0.6026	0.86	0.8051	1.46	0.9279	2.06	0.9803	2.82	0.9976
0.27	0.6064	0.87	0.8079	1.47	0.9292	2.07	0.9808	2.84	0.9977
0.28	0.6103	0.88	0.8106	1.48	0.9306	2.08	0.9812	2.86	0.9979
0.29	0.6141	0.89	0.8133	1.49	0.9319	2.09	0.9817	2.88	0.9980
0.30	0.6179	0.90	0.8159	1.50	0.9332	2.10	0.9821	2.90	0.9981
0.31	0.6217	0.91	0.8186	1.51	0.9345	2.11	0.9826	2.92	0.9983
0.32	0.6255	0.92	0.8212	1.52	0.9357	2.12	0.9830	2.94	0.9984
0.33	0.6293	0.93	0.8238	1.53	0.9370	2.13	0.9834	2.96	0.9985
0.34	0.6331	0.94	0.8264	1.54	0.9382	2.14	0.9838	2.98	0.9986
0.35	0.6368	0.95	0.8289	1.55	0.9394	2.15	0.9842	3.00	0.9987
0.36	0.6406	0.96	0.8315	1.56	0.9406	2.16	0.9846	3.05	0.9989
0.37	0.6443	0.97	0.8340	1.57	0.9418	2.17	0.9850	3.10	0.9990
0.38	0.6480	0.98	0.8365	1.58	0.9429	2.18	0.9854	3.15	0.9992
0.39	0.6517	0.99	0.8389	1.59	0.9441	2.19	0.9857	3.20	0.9993
0.40	0.6554	1.00	0.8413	1.60	0.9452	2.20	0.9861	3.25	0.9994
0.41	0.6591	1.01	0.8437	1.61	0.9463	2.21	0.9864	3.30	0.9995
0.42	0.6628	1.02	0.8461	1.62	0.9474	2.22	0.9868	3.35	0.9996
0.43	0.6664	1.03	0.8485	1.63	0.9485	2.23	0.9871	3.40	0.9997
0.44	0.6700	1.04	0.8508	1.64	0.9495	2.24	0.9875	3.45	0.9997
0.45	0.6736	1.05	0.8531	1.65	0.9505	2.25	0.9878	3.50	0.9998
0.46	0.6772	1.06	0.8554	1.66	0.9515	2.26	0.9881	3.55	0.9998
0.47	0.6808	1.07	0.8577	1.67	0.9525	2.27	0.9884	3.60	0.9998
0.48	0.6844	1.08	0.8599	1.68	0.9535	2.28	0.9887	3.65	0.9999
0.49	0.6879	1.09	0.8621	1.69	0.9545	2.29	0.9890	3.70	0.9999
0.50	0.6915	1.10	0.8643	1.70	0.9554	2.30	0.9893	3.75	0.9999
0.51	0.6950	1.11	0.8665	1.71	0.9564	2.31	0.9896	3.80	0.9999
0.52	0.6985	1.12	0.8686	1.72	0.9573	2.32	0.9898	3.85	0.9999
0.53	0.7019	1.13	0.8708	1.73	0.9582	2.33	0.9901	3.90	1.0000
0.54	0.7054	1.14	0.8729	1.74	0.9591	2.34	0.9904	3.95	1.0000
0.55	0.7088	1.15	0.8749	1.75	0.9599	2.35	0.9906	4.00	1.0000
0.56	0.7123	1.16	0.8770	1.76	0.9608	2.36	0.9909		
0.57	0.7157	1.17	0.8790	1.77	0.9616	2.37	0.9911		
0.58	0.7190	1.18	0.8810	1.78	0.9625	2.38	0.9913		
0.59	0.7224	1.19	0.8830	1.79	0.9633	2.39	0.9916		

$$\varphi(a) = \frac{1}{\sqrt{2\pi}} \int_0^a e^{-\frac{1}{2}z^2} \, dz$$

$P(0 < z < a)$

a	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990