Math 115 (Cohen,Powers) Final Exam. Thursday April 27, 2006

1. ~Consider the surface $xyz = 6$. Find the equation for the plane tangent to this surface at $(x,y,z) = (1,2,3,)$ and determine where the plane intersect the x-axis. The plane intersects the x-axis at x =

A. -2 B. -1 C. 0 D. 3 E. 5 F. 11 G. 12 H. 16

2. ~For all points on the surface of the ellipsoid $x^2 + \frac{y^2}{2} + \frac{z^2}{3} = 24$ Find the maximum of $f(x,y,z) = x + y + z$. To get full credit you must find the point(s) where the maximum occurs. Maximum =

A. 1 B. $\sqrt{2}$ C. 4 D. 6 E. 12 F. 15 G. 18 H. 24 $(x,y,x) = (2,4,6)$

3. ~The function $f(x,y) = x^3 - 3x - 2y^2$ has exactly one saddle point. Find the value of $f(x,y)$ at that point. At saddle point $f(x,y) =$ (Note you are asked the value of f at the saddle point not the coordinates of the saddle point.)

A. -3 B. -2 C. -1 D. -½ E. 0 F. $\sqrt{2}$ G. 2 H. 1½

4. ~Let S be a region inside the triangle with vertices $(x,y) = (0,0),(1,1)$ and $(0,2)$. Find the double integral $\iint_S x\, dA =$ of the function $f(x,y) = x$ over the region S.

A. 1/3 B. 1/2 C. 2 D. 3/2 E. $\sqrt{2}$ F. 0 G. -1 H. -1/2

5. ~Evaluate $\int_0^1 \int_{\sqrt{y}}^1 e^{x^3} dx\, dy.$ You may need to interchange the order of integration.

A. $\frac{e^3 - 1}{3}$ B. 3ln(3)-1 C. $\frac{e - 1}{3}$ D. $\sqrt{e}$ E. $e^{1/3}$ F. 0 G. $e\sqrt{3} - 1$ H. ln(3)

6. ~There are four coins in a box. Two are fair coins, one is weighted so the probability of heads is 3/4 and one is weighted so the probability of heads is 1/4. A coin is selected from the box at random and flipped twice. The coin lands heads both times. What is the probability it is one of the fair coins?

A. 4/9 B. 1/2 C. 2/5 D. 4/13 E. 6/17 F. 35/64 G. 1/3 H. 3/8

7. ~There are 9 ball in an jar, 5 red and 4 green. Five balls are drawn at random. What is the probability that there are more red balls than green drawn out of the jar.

A. 5/9 B. 9/14 C. 7/11 D. 2/3 E. 3/5 F. 8/13 G. 3/4 H. 7/8

8. A fair coin is tossed 8 times. What is the probability that the number of heads in the first four tosses equals the number of heads in last four tosses? (e.g. HTTHHHTT they are equal, HHTHTTHH they are not equal) (Note $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$, $2^6 = 64$, $2^7 = 128$, $2^8 = 256$)

A. $\frac{92}{256}$ B. $\frac{70}{256}$ C. $\frac{54}{256}$ D. $\frac{50}{256}$ E. $\frac{42}{256}$ F. $\frac{36}{256}$ G. $\frac{32}{256}$ H. $\frac{28}{256}$

9. Suppose X is a random vararіable distributed on the interval [0,2] with probability distribution $f(x) = (1 + x)/4$. Compute the variance of X.

A. 1/18 B. 1/12 C. 5/36 D. 4/9 E. 5/18 F. 11/36 G. 1/3

10. Let X,Y be continuous random variables with joint density function $f(x,y) = 4xy$ for $0 \leq x \leq 1$, $0 \leq y \leq 1$ and $f(x,y) = 0$ otherwise. Find the probability that $Y < 2X$.

A. 1/3 B. 7/15 C. 1/2 D. 7/8 E. 2/3 F. 5/6 G. 11/15 H. 4/5

11. Suppose the letters AABBCCC are put in a box and drawn out one a time. What is the probability that the letters will be drawn out in alpabetical order.

A. $\frac{1}{12}$ B. $\frac{1}{40}$ C. $\frac{3}{125}$ D. $\frac{2}{141}$ E. $\frac{1}{120}$ F. $\frac{1}{210}$ G. $\frac{1}{440}$ H. $\frac{1}{1024}$

12. Find the best least squares fit to the four points (x,y) = (0,1), (1,1), (1,3) and (2,3).

A. $y = -x + 6$ B. $y = x + 1$ C. $y = x + 3$ D. $y = 2x + 1$ E. $y = -2x + 2$

F. $y = -2x$ G. $y = -2x + 4$ H. $y = 2$

13. Suppose X is an exponentially distributed random variable with mean two seconds (probability density function $f(x) = (1/2)\exp(-x/2)$ for $x \geq 0$) and Y is a exponentially distributed random variable with mean four seconds. (probability density function $g(y) = (1/4)\exp(-y/4)$ for $x \geq 0$). Given the random variables X and Y are independent compute the probability that neither X nor Y occurs in a one second interval (i.e. Prob(X > 1 and Y > 1).

A. $e^{-1/4}$ B. $e^{-3/4}$ C. e^{-1} D. $e^{-3/2}$ E. 1/3 F . 1/2 G. 2/3 H. 3/4

14. The number of click of a Geiger counter is a Poisson process. On the average there is one click per minute. The Geiger counter is turned on for one minute. Given that there are at most three clicks what is the expected number of clicks?

A. $(8/3)e^{-1}$ B. $7e^{-2}$ C. $3e^{-2}$ D. 3/4 E. 11/12 F. $1 - e^{-2}$ G. 9/10 H. 15/16

15.~For each system of equations determine the number of solutions.

I.
$$\begin{aligned} x + y + z &= 1 \\ x + 2y + 2z &= 2 \\ y + z &= 2 \end{aligned}$$

II.
$$\begin{aligned} x - y + z &= 0 \\ y + z &= 1 \\ 2y + 2z &= 2 \end{aligned}$$

III.
$$\begin{aligned} x + y + z &= 1 \\ y + z &= 1 \\ 2y + 2z &= 2 \\ z &= 1 \end{aligned}$$

IV.
$$\begin{aligned} x + y + z &= 1 \\ y + z &= 1 \\ 2y + 2z &= 1 \\ z &= 0 \end{aligned}$$

Circle one answer for each system.

I. has 0 1 ∞ solution(s).

II. has 0 1 ∞ solution(s).

III. has 0 1 ∞ solution(s).

IV. has 0 1 ∞ solution(s).

16.~Find the inverse of the matrix below and add up all the nine entries. The sum of the entries of the inverse of $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ is

A. -1 B. -1/2 C. 0 D. 1 E. 3/2 F. 5/2 G. 4 H. 7/3

17.~Consider the equations $x - y = 1$ and $Ax - By = 2$ where A and B are the numbers obtained by rolling two dice numbered 1,2,3,4,5,6. What is the probability that these eqautions have no solutions.

A. 0 B. 1/36 C. 1/18 D. 1/12 E. 1/9 F. 5/36 G. 1/6 H. 7/36

18.~Peter, Paul and their older sister Mary are playing catch. The twin boys are young and prefer to throw to Mary so they throw to her 2/3 of the time and to each other 1/3 of the time while Mary throws to Peter and Paul equally often. On the average in the long run what is the probability Mary will have the ball?

A. 1/9 B. 1/7 C. 1/5 D. 2/5 E. 3/7 F. 1/2 G. 2/3 H. 7/10

19.~The average household income in a certain area is a normally distributed with mean \$50,000 and standard deviation of \$10,000. What is the probability that four randomly selected households will have a combined income over \$230,000? Use the table of the standard normal distribution below and circle the closest answer.

(No credit will be given if you do not show how you calculated the result, what you looked up and what you did with it.(e.g. $\varphi(1.5)-\varphi(0.5)=0.9332-0.6915 \approx .24$))

A. 0.5% B. 3% C. 7% D. 12% E. 20% F. 30% G. 40% H. 50%

Answers 1.D 2.E 3.B 4.A 5.C 6.A 7.B 8.B 9.F 10.D 11.F 12.B 13.B 14.H
15. 0,∞,1,0 16.E 17.F 18.D 19.C