

Math 115 (Powers,Santos) Final Exam. Friday April 29, 2005

1. Consider the surface $x^2 + 2y^2 + 3z^4 = 6$. Find the equation for the plane tangent to this surface at $(x,y,z) = (1,1,1)$ and determine where the plane intersect the x-axis. The plane intersects the x-axis at $x =$

A. -2 B. -1 C. 0 D. 4 E. $11/3$ F. $13/3$ G. 7 H. 9

2. Find the point on the plane $2x - 2y + z = 9$ which is closest to the origin and determine the distance from the origin to the plane.

The distance from the closest point to the origin is $d = \sqrt{x^2+y^2+z^2}$ is
(No credit if you do not find the closest point.)

A. 1 B. $\sqrt{2}$ C. $\sqrt{3}$ D. 2 E. $\sqrt{5}$ F. 3 G. $\sqrt{8}$ H. 4

3. The function $f(x,y) = x^2 - 2x + 2y^3 - 3y^2$ has exactly one saddle point. Find the value of $f(x,y)$ at that point. At saddle point $f(x,y) =$
(Note you are asked the value of f at the saddle point not the coordinates of the saddle point.)

A. -3 B. -2 C. -1 D. $-1/2$ E. 0 F. $\sqrt{2}$ G. 2 H. $1/2$

4. Let S be a region inside the triangle with vertices $(x,y) = (0,0), (1,1)$ and $(2,0)$. Find the double integral of the function $f(x,y) = x$ over the region S .

$$\iint_S x \, dA =$$

A. -1 B. $-1/2$ C. 0 D. 1 E. $\sqrt{2}$ F. 3 G. 4 H. $2\sqrt{2}$

5. Evaluate $\int_0^2 \int_y^2 e^{-1/2x^2} dx dy$. You may need to interchange the order of integration.

A. $\frac{e^2 - 1}{8}$ B. $2\ln(2) - 1$ C. $1/2(1 - e)$ D. $1 - e^{-2}$ E. $\frac{e^2 - 1}{4}$ F. 0 G. $e\sqrt{2} - 1$ H. $\ln(2)$

6. An urn contains 4 red balls and 6 green balls. Four balls are selected at random. What is the probability that there are two red and two green balls in the selection?

A. $1/10$ B. $1/5$ C. $13/36$ D. $2/5$ F. $3/7$ G. $19/36$ H. $2/3$

7. A fair coin is tossed repeatedly until a heads is produced. If it is know that the coin produces a heads within the first 3 flips. What is the expected number of flips to produce the first heads?

A. 1 B. $16/15$ C. $9/8$ D. $11/7$ E. $7/5$ F. $13/7$ G. 2 H. $17/8$

8. A jar contains 10 balls, 8 are red and 2 are green. The balls are drawn out of the jar one a time without replacement. What is the probability that the second green ball will be drawn on the seventh draw?
 A. $\frac{3}{5}$ B. $\frac{1}{2}$ C. $\frac{2}{5}$ D. $\frac{3}{7}$ E. $\frac{2}{17}$ E. $\frac{1}{6}$ F. $\frac{1}{8}$ G. $\frac{2}{11}$ H. $\frac{2}{15}$
9. Let X, Y be continuous random variables with joint density function $f(x, y) = xy$ for $0 \leq x \leq 2$, $0 \leq y \leq 1$ and $f(x, y) = 0$ otherwise. Find the probability that $X > Y$.
 A. $\frac{1}{3}$ B. $\frac{7}{15}$ C. $\frac{1}{2}$ D. $\frac{7}{8}$ E. $\frac{2}{3}$ F. $\frac{5}{6}$ G. $\frac{11}{15}$ H. $\frac{4}{5}$
10. Suppose X is a continuous random variable distributed on the interval $[0, 1]$ with probability density function $f(x) = kx$ where k is a constant you should determine. Compute the variance of X ?
 A. $\frac{1}{48}$ B. $\frac{1}{24}$ C. $\frac{1}{18}$ D. $\frac{1}{12}$ E. $\frac{1}{9}$ F. $\frac{2}{7}$ G. $\frac{1}{5}$ H. $\frac{1}{4}$
11. Find the best least squares fit to the four points $(x, y) =$ page
 $(0, 4), (1, 1), (1, 3)$ and $(2, 0)$.
 A. $y = -x + 6$ B. $y = x + 2$ C. $y = 2x + 3$ D. $y = 2x + 1$ E. $y = -2x + 2$
 F. $y = -2x$ G. $y = -2x + 4$ H. $y = -3x + 4$
12. Suppose X is an exponentially distributed random variable with mean two seconds (probability density function $f(x) = (1/2)\exp(-x/2)$ for $x \geq 0$) and Y is a exponentially distributed random variable with mean five seconds (probability density function $g(y) = (1/5)\exp(-y/5)$ for $x \geq 0$). Given that the random variables X and Y are independent compute the probability that X occurs after Y (i.e. $\text{Prob}(X > Y)$). (To get credit you must set up integral
 A. 0 B. $\frac{1}{4}$ C. $\frac{1}{3}$ D. $\frac{2}{7}$ E. $\frac{1}{2}$ F. $\frac{3}{7}$ G. $\frac{2}{3}$ H. $\frac{3}{4}$
13. For each system of equations determine the number of solutions.
- | | | | |
|---|--|--|--|
| $\begin{aligned} x + y + z &= 1 \\ 2x + 2y + 2z &= 0 \end{aligned}$ | $\begin{aligned} x - y + z &= 0 \\ y + z &= 1 \\ 2y + 2z &= 2 \end{aligned}$ | $\begin{aligned} x + y + z &= 1 \\ y + z &= 1 \\ 2y + 2z &= 1 \\ z &= 1 \end{aligned}$ | $\begin{aligned} x + y + z &= 0 \\ y + z &= 0 \\ 2y + 2z &= 0 \\ z &= 0 \end{aligned}$ |
|---|--|--|--|
- Circle one answer for each system. I. has 0 1 ∞ solution(s).
 II. has 0 1 ∞ solution(s).
 III. has 0 1 ∞ solution(s).
 IV. has 0 1 ∞ solution(s).

14. Consider the equations $2x - y = 1$ and $Ax - By = 1$ where A and B are the numbers obtained by rolling two dice numbered 1,2,3,4,5,6. What is the probability that these equations have no solutions.
A. 0 B. $1/36$ C. $1/18$ D. $1/12$ E. $1/9$ F. $1/4$ G. $1/2$ H. $2/3$
15. Peter, Paul and their younger sister Mary are playing catch. The boys prefer to throw to each other so they throw to each other $2/3$ of the time and to their younger sister $1/3$ of the time while, Mary is equally likely to throw to Peter or Paul. On the average in the long run what is the probability Mary will have the ball?
A. $1/9$ B. $2/9$ C. $1/4$ D. $2/5$ E. $5/18$ F. $11/36$ G. $1/3$ H. $7/18$
16. The number of click of a Geiger counter is a Poisson process. On the average there are three clicks per minute. Given that during a twenty second interval there are at most three clicks what is the probability that there were no clicks during that twenty second interval?
A. e^{-1} B. e^{-3} C. $1 - e^{-1}$ D. $3/8$ E. $1/2$ F. $1 - 3e^{-3}$ G. $2/3$ H. 1
17. An unfair coin has a probability of $2/3$ for coming up heads and $1/3$ of coming up tails. The coin is tossed 450 times. What is the probability that there between 295 and 305 heads? Use the table of the standard normal distribution below and circle the closest answer.
A. 1% B. 5% C. 10% D. 25% E. 40% F. 50% G. 65% H. 85%
18. A simple model for a computer company divides the company into two sectors hardware and software. To produce a \$1 worth of software requires \$0.40 of software and \$0.20 of hardware. And to produce a \$1 worth of hardware requires \$0.30 of software and \$0.20 of hardware To meet a demand of \$9 of software and \$4 of hardware how should the production levels be set in dollars. (Software, Hardware) =
A. (20,10) B. (20,20) C. (30,70) D. (50,50) E. (25,45) F. (10,5) G. (12,8) H. (12,8)
19. A gardener intends to grow corn and soy beans in a 5 acre field. See relevant data below.
- | | corn | soy beans | Total acres = 5 |
|------------------------|-------|-----------|------------------|
| planting cost per acre | \$200 | \$100 | |
| water needs per acre | 3,000 | 2,000 | gallons per week |
| profit | \$400 | \$300 | |
- If there is \$700 available for planting cost and 11,000 gallons per week for watering, what will be the maximam profit?
A. \$1300 B. \$1400 C. \$1500 D. \$1600 E. \$1700 F. \$1800 G. \$1900 H. \$20000

Answers. HFCD DFDH DCGD (13. 0,∞,0,1) CCDE AD

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Table 1: Areas for $\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z e^{-x^2/2} dx$