MATH 115 FINAL EXAM. THURSDAY MAY 6, 2004

1. Consider the surface $x^6 + 2y^4 + 4z^2 = 7$. Find the equation for the plane tangent to this surface at $(x, y, z) = (1, 1, 1,)$ and determine where the plane intersect the x-axis. The plane intersects the x-axis at $x =$

$A. -3 \quad B. -1 \quad C.0 \quad D.4 \quad E.11/3 \quad F.13/3 \quad G. -7/3 \quad H.8$

2. Find the point on the plane $2x + 2y + z = 9$ which is closest to the origin. The closest point has coordinates $(x, y, z) =$

$A.(1, 1, 5) \quad B.(\frac{1}{2}, \frac{1}{2}, 7) \quad C.(2, 2, 1) \quad D.(\frac{3}{2}, \frac{3}{2}, 3) \quad E.(-1, 1, 9) F.(0, 0, 9)$
$G.(\frac{9}{5}, \frac{9}{5}, \frac{9}{5}) \quad H.(1, 2, 3)$

3. Let $r(x, y) = \sqrt{x^2 + y^2}$. Note $r(3, 4) = 5$. Using differentials to approximate $r(3.1, 3.9)$ one gets

$A.5 - \frac{1}{25} \quad B.5 - \frac{1}{50} \quad C.5 + \frac{1}{50} \quad D.5 + \frac{1}{25} \quad E.5 - \frac{1}{20} \quad F.5$
$G.5 + \frac{1}{30} \quad H.5 - \frac{1}{80}$

4. The function $f(x, y) = x^2 - 4x + y^3 - 3y$ has two critical points. Find them and deteremine their type.

$A.$(rel min at $x = 2, y = +1$, rel min at $x = 2, y = -1$)
$B.$(rel min at $x = 2, y = +1$, saddle at $x = 2, y = -1$)
$C.$(rel min at $x = 2, y = +1$, rel max at $x = 2, y = -1$)
$D.$(saddle at $x = 2, y = +1$, rel min at $x = 2, y = -1$)
$E.$(saddle at $x = 2, y = +1$, saddle at $x = 2, y = -1$)
$F.$(saddle at $x = 2, y = +1$, rel max at $x = 2, y = -1$)
$G.$(rel max at $x = 2, y = +1$, rel min at $x = 2, y = -1$)
$H.$(rel max at $x = 2, y = +1$, saddle at $x = 2, y = -1$)

5. Evaluate

$$\int_0^{\frac{1}{2}} \int_{4y^2}^1 y e^{x^2} \, dx \, dy.$$

$A.\frac{e^2 - 1}{8} \quad B.2\ln(2) - 1 \quad C.\frac{1}{2}(e + 1) \quad D.2 + e^{-2} \quad E.\frac{e - 1}{16} \quad F.0 \quad G.e\sqrt{2} \quad H.\ln(2)$

6. Three fair dice numbered 1-6 are tossed. What is the probability all three dice show a different number?

$A.1/3 \quad B.5/9 \quad C.13/36 \quad D.1/2 \quad F.17/36 \quad G.109/216 \quad H.2/3$

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7. There are three coins. Coin A produces heads 2/3 of the time, coin B is fair, coin C produces heads 1/3 of the time. A coin is selected at random and tossed twice. It produces a heads on the first flip and a tails on the second flip. What is the probability the selected coin was the fair coin, coin B.

$A.9/25 \quad B.19/36 \quad C.1/3 \quad D.2/5 \quad E.4/7 \quad F.17/36 \quad G.5/9 \quad H.4/9$

8. A jar contains 6 red balls and 4 green balls. If three balls are selected at random (without replacement) what is the probability that there are more red balls than green balls?

$A.3/10 \quad B.1/2 \quad C.3/5 \quad D.6/11 \quad E.2/3 \quad E.41/60 \quad F.7/11 \quad G.83/120 \quad H.7/13$

9. In the world series of foosball, a three-game match is played, and the player who wins the most games is the champion. The probability of Player A winning any given game against Player B is always 3/5. What is the probability that Player A will be the champion? (You may assume all three games are played even if one player wins the first two games.)

$$A.\frac{12}{15} \quad B.\frac{67}{125} \quad C.\frac{14}{25} \quad D.\frac{3}{5} \quad E.\frac{76}{125} \quad F.\frac{81}{125} \quad G.\frac{93}{125} \quad H.\frac{4}{5}$$

10. Six different pairs of socks (red, blue, gray, white, purple and green) go to the laundry (12 socks in all) and 9 come back. What is the expected number of pairs of socks that come back? Expected number of pairs of socks that come back =

$A.5/2 \quad B.11/3 \quad C.3 \quad D.22/7 \quad E.34/11 \quad F.36/11 \quad G.40/11 \quad H.4$

11. Find the best least squares fit to the four points $(x, y) = (0, 1)$, $(1, 2)(1, 4)$ and $(2, 5)$.

$A.y = -x + 6 \quad B.y = x + 2 \quad C.y = x + 3 \quad D.y = 2x + 1 \quad E.y = 2x + 3$
$F.y = 3x \quad G.y = 3x + 1 \quad H.y = 3x - 1$

12. Suppose X is a continuous random variable distributed on the interval $[0, 4]$ with probablility distribution $f(x) = x/8$. Compute the conditional probality that X lies between 2 and 3 given that X lies between 2 and 4. (i.e. Compute $\Pr(2 < X < 3|2 < X < 4))$.$Pr(2 < X < 3|2 < X < 4) =$

$A.1/4 \quad B.1/3 \quad C.5/12 \quad D.11/24 \quad E.1/2 \quad F.7/16 \quad G.7/12 \quad H.9/16$

13. A point (x, y) is chosen at random on a rectangle 3 feet by 5 feet. What is the probability that the two points are within two feet of each other? (i.e. compute $\Pr(\ |x - y| < 2$ feet). Prob =

$A.1/3 \quad B.7/15 \quad C.1/2 \quad D.8/15 \quad E.2/3 \quad F.7/10 \quad G.11/15 \quad H.4/5$

14. Suppose X is an exponentially distributed random variable with mean two seconds (probability density function $f(x) = (1/2)\exp(-x/2)$ for $x \geq 0$) and Y is

a exponentially distributed random variable with mean four seconds (probability density function $g(y) = (1/4)\exp(-y/4)$ for $x \geq 0$). Given that the random variables X and Y are independent compute the probability that X occurs after Y (i.e. Prob$(X > Y)$). (To get credit you must set up and evaluate the double integral.)

$A.0 \quad B.1/4 \quad C.1/3 \quad D.2/5 \quad E.1/2 \quad F.3/5 \quad G.2/3 \quad H.3/4$

15. For what values of k does the following matrix

$$\begin{bmatrix} 1 & 0 & k \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

have an inverse?

$A.$for all values of k $\quad B.$for no values of k

$C.$only for $k \neq 0$ $\quad D.$only for $k \neq -1 E.$ only for $k \neq 1$

$F.$ only for $k \neq 2$ $\quad G.$only for $k = 1 H.$ only for $k = 2$

16. Three people A, B and C are playing catch. The probabilities each will throw to the others is: Prob(A$\rightarrow B$) $= 1/2$ Prob(A$\rightarrow C$) $= 1/2$ Prob(B$\rightarrow A$) $= 1/4$ Prob(B$\rightarrow C$) $= 3/4$ Prob(C$\rightarrow A$) $= 1/2$ Prob(C$\rightarrow B$) $= 1/2$. What is the probability that A will have the ball in the long run?

$A.1/9 \quad B.2/9 \quad C.1/4 \quad D.2/5 \quad E.5/18 \quad F.11/36 \quad G.1/3 \quad H.7/18$

17. The number of transmission errors in any given time interval is a Poisson process. The average number of transmission errors is one error every two seconds. What is the probability there will be three or more transmission errors in a four second interval?

$A.e^{-2} \quad B.1 - 2e^{-2} \quad C.1 - 5e^{-2} \quad D.1 - 2/e \quad E.5/e \quad F.1 - 4e^{-2} \quad G.2/3 \quad H.1/e$

18. A simple model of the ecomomy divides the ecomomy into two sectors agriculture and manufacturing. To produce \$100 of agricultural products is requires \$20 of agricultural products and \$20 of manufactured products and to produce \$100 of manufactured products it requires \$40 of agricultural products and \$20 of manufactured products. To produce \$2 of agricultural products and \$3 of manufactured products for outside demand, how should the production levels be set in dollars. (Agriculture, Manufacturing) =

$A.(2.50, 5.00) \quad B.(2.50, 7.00) \quad C.(3.00, 7.00) \quad D.(5.00, 5.00) \quad E.(5.50, 4.50)$

$F.(5.50, 6.00) \quad G.(6.00, 4.00) \quad H.(6.50, 4.50)$

19. The faces of a die are numbered 1,1,2,2,3,3 so there is a 1/3 probability of producing a 1, 2 or 3. The die is tossed 600 times and the sum of the numbers is recorded. What is the probability that the sum is between 1200 and 1230. (Hint. The variance for the sum of the tosses is 600 times the variance for one toss.) Use the table of the standard normal distribution below and circle the closest answer. (No credit will be given if you do not show how you calculated the result, what you looked up and what you did with it.(e.g. $\phi(1.5) - \phi(0.5) = 0.9332 - 0.6915 \approx .24$))

$A.5\% \quad B.15\% \quad C.25\% \quad D.35\% \quad E.45\% \quad F.55\% \quad G.65\% \quad H.75\%$

Answers ECCB EBAE FFDC ECAE CDE