Math 115 **Final Exam** **Spring 2003**

ANSWERS AT THE END

1. X is a continuous random variable on the interval [0,1] whose density function is of the form $k(1-x)$ for some constant k. What is Var(X)?

   A. 1/2 B. 1/3 C. 1/6 D. 1/9 E. 1/12 F. 1/18 G. $\sqrt{2}/6$ H. $\sqrt{3}/6$

2. A Geiger counter clicks, on the average, every 20 seconds. (The number of clicks is a Poisson process.) It is known that during one particular minute the counter clicked at most 3 times. What is the pobability that the number of clicks during that minute was at least 2?

   A. 4/13 B. 9/13 C. 12/13 D. 0
   E. $4e^{-3}$ F. $1-4e^{-3}$ G. $13e^{-3}$ H. $1-13e^{-3}$

3. Find
$$\frac{\partial^2}{\partial x \partial y}\Big(x + xy - 5x^3 + y\ln(x^2+1)\Big)$$
   A. $1+\frac{1}{x^2+1}$ B. $\big(x+\ln(x^2+1)\big)\big(1+y-15x^2+\frac{2xy}{x^2+1}\big)$ C. 0 D. $1-\frac{2}{x^2+1}$
   E. $\frac{2y}{x^2+1}-\frac{4x^2y}{(x^2+1)^2}-30x$ F. $\frac{y}{x^2+1}-\frac{2xy}{(x^2+1)^2}-30x$ G. $\frac{x^2+2x+1}{x^2+1}$ H. $\frac{x^2+3}{x^2+1}$

4. If the measurements of a and b to the nearest 1/10 of an inch are $a=10$ inches and $b=16$ inches then the maximum percentage error in calculating the area $A=\pi ab$ of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is closest to:

   A. $2.6\pi\%$ B. $160\pi\%$ C. $26\pi\%$ D. 13/8%
   E. $1.60\pi\%$ F. $\pi\%$ G. 356/256000% H. 50%

5. A simple economy consists of two sectors, agriculture and manufacturing. The input–output matrix is
$$A := \begin{bmatrix} .4 & .2 \\ .1 & .3 \end{bmatrix}.$$
   How many units (in the form [agriculture,manufacturing]) should be produced by each sector to meet the consumer demand of 20 units agriculture and 12 units manufacturing?

   A. $[25,17]$ B. $[20,12]$ C. $[21,13]$ D. $[40,24]$
   E. $[41,23]$ F. $[80,48]$ G. $[81,49]$ H. $[82,50]$

6. The maximum value of $P = 2y - 7x + 15$ subject to the constraints

   - $y \leq 5$
   - $y \leq 4x - 7$
   - $4y \geq x + 2$
   - $x + y \leq 8$

   is:

   A. 1 B. 2 C. 3 D. 4 E. 5 F. 6 G. 7
   H. not obtained anywhere

7. Let $w = \ln(1 + \frac{x^2}{2}) - \arctan(x)$ and $x = 3e^u \cos(v) + v$. Find $\frac{\partial w}{\partial u}$ at $u = v = 0$.
   A. $\frac{147}{110}$ B. $\frac{196}{110}$ C. 0 D. $\frac{49}{110}$ E. $\frac{3}{10}$ F. 3 G. $\frac{4}{10}$ H. undefined

8. At which one of the following points is the tangent plane to the surface $xy + yz + zx - x - z^2 = 0$ parallel to the xy-plane.
   A. $(\frac{1}{2}, 1, \frac{1}{2})$ B. $(-1, 0, 0)$ C. $(1, 1, 1)$ D. $(-\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$
   E. $(1, 1, 0)$ F. $(\frac{1}{2}, \frac{1}{2}, -\frac{1}{2})$ G. $(0, -1, 0)$ H. $(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2})$

9. Consider the following systems of linear equations:

   System 1:
   $$\begin{aligned} 4w - 3x + 8y - 9z &= 7 \\ 2x + 7y - 5z &= 9 \\ 3y - 6z &= 12 \\ y - 2z &= 4 \end{aligned}$$

   System 2:
   $$\begin{aligned} 4w - 3x + 8y - 9z &= 7 \\ 2x + 7y - 5z &= 9 \\ 3y - 6z &= 11 \\ y - 2z &= 4 \end{aligned}$$

   System 3:
   $$\begin{aligned} 4w - 3x + 8y - 9z &= 7 \\ 2x + 7y - 5z &= 9 \\ 3y - 7z &= 12 \\ y - 2z &= 4 \end{aligned}$$

   Which of these systems have infinitely many solutions?

   A. 1, 2, 3 B. 1, 2 but not 3 C. 1, 3 but not 2 D. 2, 3 but not 1
   E. 1 but not 2, 3 F. 2 but not 1, 3 G. 3 but not 1, 2 H. None

10. Peter, Paul, and Mary are playing catch. The boys are twice as likely to throw to Mary as to each other, while Mary is equally likely to throw to Peter or to Paul. On the average, for what portion of the play time will Mary have the ball?

A. 2/5 B. 3/5 C. 3/10 D. 3/7 E. 4/7 F. 1/3 G. 2/3 H. 5/6

11. Find the minimum value of the function $f(x, y) = 3 - 3x - 4y$ when subject to the constraint $x^2 + y^2 = 1$.

A. -8 B. -2 C. $-9/5$ D. $-8/5$ E. $9/5$ F. 3 G. $22/5$ H. 8

12. Evaluate :

$$\int_0^8 \int_{x^{1/3}}^2 \frac{1}{y^4 + 1} dy dx$$

A. $\frac{1}{4} \ln 17$ B. $\frac{1}{9}$ C. $\ln \frac{1}{4}$ D. $\frac{17}{4}$ E. $\tan^{-1} 4$ F. $\ln \sqrt{17}$ G. $\frac{1}{65}$ H. 0

13. For which value of k does the following system have at least one solution?

$$\begin{array}{rrrl} 2x & -y & +3z & = 7 \\ x & +y & +2z & = 4 \\ x & -5y & & = k \end{array}$$

A. 0 B. 1 C. 2 D. 3 E. 4 F. 6 G. no k H. all k

14. The scores on a math test are approximately normally distributed with $\mu = 80$ and $\sigma = 8$. Which percentage of students scored 90 points or higher?

A. 39.44 B. 27.04 C. 10.56 D. 8.51 E. 7.21 F. 5.67 G. 0.02 H. 0

15. It has been determined that at a certain intersection cars arriving from the west go straight 10% of the time, turn left 70% of the time, and turn right 20% of the time. It is also known that 80% of drivers use their turn signals regularly (you can assume always) while 20% use them rarely (you can assume never). You, who are heading into the intersection from the west, are sitting behind a driver who does not have his turn signal on. What is the probability that he is turning left?

A. 0.2 B. 0.3 C. 0.4 D. 0.5 E. 0.6 F. 0.7 G. 0.8 H. 0.9

16. A bridge hand consists of 13 card from a standard 52-card deck. Find the probability that a bridge hand contains no aces?

A. $\frac{1}{13}$ B. $\frac{4}{13}$ C. $\frac{48!}{9!}\frac{13!}{52!}$ D. 0 E. $\frac{4!}{52!}$ F. $1 - \frac{13!}{52!}$ G. $\frac{3}{4}$ H. $\frac{48!39!}{52!35!}$

17. Suppose a committee of 8 people is selected in a random manner from 15 people. Determine the probability that two particular people, A and B, will both be selected.

A. $\frac{1}{15}$ B. $\frac{8!}{15!}$ C. $\frac{4}{15}$ D. $\frac{13!}{6!7!}\frac{8!}{15!}$ E. $\frac{7!}{15!}$ F. $\frac{13!}{6!8!}\frac{7!}{15!}$ G. $\frac{13!}{15!}$ H. $\frac{8!}{6!15}$

18. Consider three events A, B and C. Assume we know $Pr(A) = .4$, $Pr(B) = .4$, $Pr(C) = .5$, $Pr(A \cap B \cap C) = .1$, $Pr(A|C) = .4$, $Pr(B|C) = .4$, and $Pr(A|B) = .5$. Then $Pr(A \cup B \cup C)$ is:

A. 0 B. 0.2 C. 0.3 D. 0.4 E. 0.5 F. 0.6 G. 0.8 H. 1

Answers

1. F 1/18
2. B. 9/13
3. G. $\frac{x^2+2x+1}{x^2+1}$
4. D. 13/8%
5 . E. $[41, 23]$
6. D. 4
7. A. $\frac{147}{110}$
8. D. $(-\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$
9. E. 1 but not 2, 3
10. A. 2/5
11. B. -2
12. A. $\frac{1}{4}\ln 17$
13. C. 2
14. C. 10.56
15. D. 0.5
16. H. $\frac{48!39!}{52!35!}$
17. C. $\frac{4}{15}$
18. G. 0.8

P(0 <z< a)

a	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990