Math 115 **Makeup Final Exam** **Spring 2003**

ANSWERS AT THE END

1. X is a continuous random variable on the interval [0,1] whose density function is of the form kx^2 for some constant k. What is Var(X)?

   A. $\frac{1}{80}$ B. $\frac{1}{40}$ C. $\frac{3}{80}$ D. $\frac{1}{20}$ E. $\frac{5}{80}$ F. $\frac{3}{40}$ G. $\sqrt{\frac{1}{80}}$ H. $\sqrt{\frac{3}{80}}$

2. A Geiger counter clicks, on the average, every 15 seconds. (The number of clicks is a Poisson process.) Let X be the total number of clicks of three such geiger counters in a minute. How many clicks per minute is one standard deviation of X.

   A. 12 B. 45 C. $\sqrt{45}$ D. 8 E. $\sqrt{12}$ F. 6 G. $\sqrt{6}$ H. $\sqrt{8}$

3. Find $\dfrac{\partial^2}{\partial x \partial y}\left(\dfrac{x}{y} + y^2 + x^y\right)$

   A. $\ln(y) + x^y \ln(x)$ B. $-\frac{1}{y^2} + x^y \ln(x)$ C. $-\frac{1}{y^2} + x^{y-1}(yln(x) + 1)$
   D. $x \ln(y) + x^{y-1} ln(x)$ E. $\frac{x}{y^3} + x^{y-1}(yln(x) + 1) + 2$ F. $-\frac{1}{y^2} + x^{y-1}$
   G. $\frac{2}{y^2} + x^y \ln(x)$ H. 0

4. If the measurements of a and b to the nearest 1/10 of an inch are $a = 5$ inches and $b = 6$ inches then the maximum percentage error in calculating the area $A = \pi ab$ of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is closest to:

   A. $30\pi\%$ B. $1.1\pi\%$ C. $18\pi\%$ D. $11\pi\%$
   E. $3\pi\%$ F. $61/9600\%$ G. $10/3\%$ H. $11/3\%$

5. The waiting time for an elevator is an exponentially distributed random variable with mean 3 minutes. When your guest arrives at your floor you ask if he had to wait longer than 6 minutes for the elevator. He says no. What is the probability that he had to wait at least 3 minutes?

   A. 1/2 B. $(e-1)/(e-2)$ C. $(e^{-1})/(e^{-2})$ D. $(e^{-2})/(e^{-1})$
   E. e^{-2} F. $e^{-1} - e^{-2}$ G. $(e^{-1} - e^{-2})/(e^{-2} - 1)$ H. $(e^{-1} - e^{-2})/(1 - e^{-2})$

6. A computer manufacturer assembles two kinds of computers, A and B. A computer of type A requires 6 minutes of cleaning and 2 minutes of testing, whereas a B type computer requires 3 minutes of cleaning and 4 minutes of testing. The cleaning facility is available at most 75 minutes per day and the testing facility is available at most 40 minutes per day. The company can assemble at most 20 computers per day. The profit is \$2 on each A-computer and \$5 on each B-computer. Let x be the number of A-computers and y the number of B-computers assembled per day. Find the values of x and y that maximize the profit.

A. $(0, 10)$ B. $(0, 20)$ C. $(0, 25)$ D. $(0, 0)$
E. $(25/2, 0)$ F. $(20, 0)$ G. $(10, 5)$ H. $(5, 15)$

7. Let $w = \ln(1 + \frac{x^2}{2}) - \arctan(x)$ and $x = 3e^u \cos(v) + v$. Find $\frac{\partial w}{\partial v}$ at $u = v = 0$.

A. $\frac{147}{110}$ B. $\frac{196}{110}$ C. 0 D. $\frac{49}{110}$ E. $\frac{3}{10}$ F. 3 G. $\frac{4}{10}$ H. undefined

8. Find a point on the surface $y^2 + (y + x)^2 + (z - y)^2 = 25$ where the tangent plane is parallel to the xz-plane.

A. $(5, 0, 0)$ B. $(-5, 0, 0)$ C. $(2, 2, \frac{1}{5})$ D. $(-5, 5, 5)$
E. $(0, 3, 0)$ F. $(2, 2, -5)$ G. $(0, 0, 3)$ H. $(0, 0, 5)$

9. Peter, Paul, and Mary are playing catch. The boys throw to Mary 3/4 of the time, while Mary is equally likely to throw to Peter or to Paul. On the average, for what portion of the play time will Mary have the ball?

A. 2/5 B. 3/5 C. 3/10 D. 3/7 E. 4/7 F. 1/3 G. 2/3 H. 5/6

10. Find the minimum value of the function $f(x, y) = 2 - 3x - 4y$ when subject to the constraint $x^2 + y^2 = 1$.

A. -9 B. -3 C. $-14/5$ D. $-13/5$ E. 4/5 F. 2 G. 17/5 H. 7

11. Evaluate :

$$\int_0^1 \int_{2y}^2 4\cos(x^2)dxdy$$

A. 1 B. $\cos^2 2$ C. $\frac{1}{2}\sin\frac{1}{2}$ D. 4 E. $\cos 4$ F. 18 G. $\sin 4$ H. 0

12. For which value of k does the following system have one and only one solution?

$$\begin{array}{rrrl} 2x & -y & +z & =5 \\ x & +y & +2z & =1 \\ x & -y & & =k \end{array}$$

A. 0 B. 1 C. 2 D. 3 E. 4 F. 5 G. no k H. all k

13. $A = \begin{bmatrix} 0 & 3 & -2 \\ 2 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix}$

The sum of the entries in the third row of A^{-1} equals:

A. -2 B. 0 C. 2 D. 4 E. 8 F. 10 G. -4 H. -8

14. A chest of drawers has three drawers: the top, the middle, and the bottom. The top drawer contains 3 black and 2 white pairs of socks, the middle drawer has 2 black and 4 white pairs of socks, and the bottom drawer has 4 black and 2 white pairs of socks. A drawer is selected at random (any drawer is equally likely to be chosen) and a pair of socks from the drawer is chosen at random. Given that the pair is white, what is the probability that it came from the middle drawer?

A. 1/8 B. 2/9 C. 7/15 D. 1/2 E. 2/15 F. 10/21 G. 15/32 H. 5/9

15. A bridge hand consists of 13 card from a standard 52-card deck. Find the probability that a bridge hand contains all four aces?

A. $\frac{1}{13}$ B. $\frac{4}{13}$ C. $\frac{48!}{9!}\frac{13!}{52!}$ D. 0 E. $\frac{4!}{52!}$ F. $1-\frac{13!}{52!}$ G. $\frac{3}{4}$ H. $\frac{48!}{52!}$

16. Consider three events E, F and G. Assume that they have the following probabilities: $Pr(E) = \frac{1}{2}$, $Pr(F) = \frac{9}{20}$, and $Pr(G) = \frac{2}{5}$. Assume that we also know the following conditional probabilities $Pr(E|F) = \frac{2}{3}$, $Pr(E|G) = \frac{5}{8}$, $Pr(F|G) = \frac{3}{8}$, and $Pr((E \cap G)|F) = \frac{2}{9}$. Then $Pr(F \cup (E \cap G))$ is (There may be more information than is needed to solve the problem):

(a) $\frac{1}{4}$ (b) $\frac{1}{10}$ (c) $\frac{16}{25}$ (d) $\frac{3}{5}$ (e) $\frac{2}{9}$ (f) $\frac{7}{10}$ (g) $\frac{3}{4}$ (h) $\frac{19}{20}$

17. An IQ test is scaled to give a mean score of 100 with a standard deviation of 20. (Assume that the scores are normallly distibuted.) Children having IQs of less than 80 or greater than 145 are deemed to need special attention. Given a population of 2000 children, what will be the expected demand for these additional services?

    A. less than 100 B. 100 to 165 C. 170 to 220 D. 225 to 265
    E. 270 to 310 F. 315 to 355 G. 360 to 400 H. more than 400

18. A simple economy consists of two sectors, agriculture and tourism. The input–output matrix is

$$A := \begin{bmatrix} .2 & .2 \\ .3 & .3 \end{bmatrix}.$$

    How many units (in the form [agriculture,fishing]) should be produced by each sector to meet the consumer demand of 20 units agriculture and 10 units fishing?

    A. $[6, 9]$ B. $[14, 1]$ C. $[20, 10]$ D. $[26, 19]$
    E. $[30, 30]$ F. $[32, 28]$ G. $[34, 11]$ H. $[40, 20]$

Answers

1. C
2. E
3. C
4. H
5. H
6. A
7. D
8. D
9. D
10. B: -3
11. G
12. G
13. F
14. F
15. C
16. D
17. F
18. F

P(0 <z< a)

a	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990