MATH 115 - FINAL EXAM

May 1, 2003

Name _____

Student no. (from ID)_____

Rec. Day & time _____

Teaching Assistant _____

This is a Multiple choice, closed book, no calculator exam. You may use a $5'' \times 8''$ card. Show all your work.

PUT YOUR ANSWERS ON THE ANSWER SHEET (page 10). Next to the number for each problem circle your letter answer (for example if your answer to problem 12 is C circle the letter C next to 12 on the answer sheet). Make sure that you give an answer for each question and that you have circled the letter that you intended to!

There is a table for the area under the standard normal distribution at the end of the exam that you may need.

1. Find

$$\frac{\partial^2}{\partial x \partial y}\Big(x + xy - 5x^3 + y\ln(x^2+1)\Big)$$

A. $1+\frac{1}{x^2+1}$ B. $\big(x+\ln(x^2+1)\big)\big(1+y-15x^2+\frac{2xy}{x^2+1}\big)$ C. 0 D. $1-\frac{2}{x^2+1}$
E. $\frac{y}{x^2+1}-\frac{2xy}{(x^2+1)^2}-30x$ F. $\frac{2y}{x^2+1}-\frac{4x^2y}{(x^2+1)^2}-30x$ G. $\frac{x^2+3}{x^2+1}$ H. $\frac{x^2+2x+1}{x^2+1}$

2 Consider three events A, B and C. Assume we know $Pr(A) = .4$, $Pr(B) = .4$, $Pr(C) = .5$, $Pr(A \cap B \cap C) = .1$, $Pr(A|C) = .4$, $Pr(B|C) = .4$, and $Pr(A|B) = .5$. Then $Pr(A \cup B \cup C)$ is:

A. 0 B. 0.2 C. 0.4 D. 0.6 E. 0.8 F. 0.3 G. 0.5 H. 1

3. Consider the following systems of linear equations:

System 1:
$$\begin{aligned} 4w - 3x + 8y - 9z &= 7 \\ 2x + 7y - 5z &= 9 \\ 3y - 6z &= 12 \\ y - 2z &= 4 \end{aligned}$$

System 2:
$$\begin{aligned} 4w - 3x + 8y - 9z &= 7 \\ 2x + 7y - 5z &= 9 \\ 3y - 6z &= 11 \\ y - 2z &= 4 \end{aligned}$$

System 3:
$$\begin{aligned} 4w - 3x + 8y - 9z &= 7 \\ 2x + 7y - 5z &= 9 \\ 3y - 7z &= 12 \\ y - 2z &= 4 \end{aligned}$$

Which of these systems have infinitely many solutions?

A. 1, 2, 3 B. 2, 3 but not 1 C. 1, 3 but not 2 D. 1, 2 but not 3
E. 3 but not 1, 2 F. 2 but not 1, 3 G. 1 but not 2, 3 H. None

4. If the measurements of a and b to the nearest $1/10$ of an inch are $a = 10$ inches and $b = 16$ inches then the maximum percentage error in calculating the area $A = \pi ab$ of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is closest to:

A. $2.6\pi\%$ B. $26\pi\%$ C. $160\pi\%$ D. $1.60\pi\%$
E. $\pi\%$ F. $13/8\%$ G. $356/256000\%$ H. 50%

5. The maximum value of $P = 2y - 7x + 15$ subject to the constraints

   - $y \leq 5$
   - $y \leq 4x - 7$
   - $4y \geq x + 2$
   - $x + y \leq 8$

   is:

   A. 2 B. 4 C. 6 D. 1 E. 3 F. 5 G. 7
   H. not obtained anywhere

6. For which value of k does the following system have at least one solution?

$$\begin{array}{rrrl} 2x & -y & +3z & = 7 \\ x & +y & +2z & = 4 \\ x & -5y & & = k \end{array}$$

A. 0 B. 2 C. 4 D. 6 E. 1 F. 3 G. no k H. all k

7. X is a continuous random variable on the interval [0,1] whose density function is of the form $k(1-x)$ for some constant k. What is Var(X)?

   A. 1/6 B. 1/12 C. 1/2 D. 1/9 E. 1/3 F. $\sqrt{3}/6$ G. $\sqrt{2}/6$ H. 1/18

8. A bridge hand consists of 13 card from a standard 52-card deck. Find the probability that a bridge hand contains no aces?

   A. $\frac{48!39!}{52!35!}$ B. $\frac{48!}{9!}\frac{13!}{52!}$ C. $\frac{4}{13}$ D. $\frac{4!}{52!}$ E. 0 F. $1-\frac{13!}{52!}$ G. $\frac{3}{4}$ H. $\frac{1}{13}$

9. At which one of the following points is the tangent plane to the surface $xy + yz + zx - x - z^2 = 0$ parallel to the xy-plane.

A. $(-1, 0, 0)$ B. $(\frac{1}{2}, 1, \frac{1}{2})$ C. $(-\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ D. $(1, 1, 1)$
E. $(1, 1, 0)$ F. $(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2})$ G. $(0, -1, 0)$ H. $(\frac{1}{2}, \frac{1}{2}, -\frac{1}{2})$

10. Find the minimum value of the function $f(x, y) = 3 - 3x - 4y$ when subject to the constraint $x^2 + y^2 = 1$.

A. -2 B. -8 C. $-9/5$ D. $-8/5$ E. 3 F. 8 G. 22/5 H. 9/5

11. Suppose a committee of 8 people is selected in a random manner from 15 people. Determine the probability that two particular people, A and B, will both be selected.

A. $\frac{1}{15}$ B. $\frac{8!}{15!}$ C. $\frac{7!}{15!}$ D. $\frac{13!}{6!8!}\frac{7!}{15!}$ E. $\frac{4}{15}$ F. $\frac{13!}{6!7!}\frac{8!}{15!}$ G. $\frac{13!}{15!}$ H. $\frac{8!}{6!15}$

12 A Geiger counter clicks, on the average, every 20 seconds. (The number of clicks is a Poisson process.) It is known that during one particular minute the counter clicked at most 3 times. What is the pobability that the number of clicks during that minute was at least 2?

A. $4e^{-3}$ B. $1-4e^{-3}$ C. $13e^{-3}$ D. $1-13e^{-3}$
E. 4/13 F. 9/13 G. 12/13 H. 0

13. The scores on a math test are approximately normally distributed with $\mu = 80$ and $\sigma = 8$. Which percentage of students scored 90 points or higher?

    A. 0 B. 0.02 C. 5.67 D. 7.21 E. 8.51 F. 10.56 G. 27.04 H. 39.44

14. Peter, Paul, and Mary are playing catch. The boys are twice as likely to throw to Mary as to each other, while Mary is equally likely to throw to Peter or to Paul. On the average, for what portion of the play time will Mary have the ball?

    A. 1/3 B. 2/3 C. 2/5 D. 3/5 E. 5/6 F. 3/7 G. 4/7 H. 3/10

15. Let $w = \ln(1 + \frac{x^2}{2}) - arctan(x)$ and $x = 3e^u \cos(v) + v$. Find $\frac{\partial w}{\partial u}$ at $u = v = 0$.

A. $\frac{49}{110}$ B. $\frac{196}{110}$ C. $\frac{4}{10}$ D. $\frac{147}{110}$ E. $\frac{3}{10}$ F. 3 G. 0 H. undefined

16. A simple economy consists of two sectors, agriculture and manufacturing. The input–output matrix is

$$A := \begin{bmatrix} .4 & .2 \\ .1 & .3 \end{bmatrix}.$$

How many units (in the form [agriculture,manufacturing]) should be produced by each sector to meet the consumer demand of 20 units agriculture and 12 units manufacturing?

A. $[20, 12]$ B. $[21, 13]$ C. $[25, 17]$ D. $[40, 24]$
E. $[41, 23]$ F. $[80, 48]$ G. $[81, 49]$ H. $[82, 50]$

17. Evaluate :

$$\int_0^8 \int_{x^{1/3}}^2 \frac{1}{y^4+1} dy dx$$

A. 0 B. $\frac{1}{9}$ C. $\frac{17}{4}$ D. $\frac{1}{4}\ln 17$ E. $\tan^{-1} 4$ F. $\ln\sqrt{17}$ G. $\frac{1}{65}$ H. $\ln\frac{1}{4}$

18. It has been determined that at a certain intersection cars arriving from the west go straight 10% of the time, turn left 70% of the time, and turn right 20% of the time. It is also known that 80% of drivers use their turn signals regularly (you can assume always) while 20% use them rarely (you can assume never). You, who are heading into the intersection from the west, are sitting behind a driver who does not have his turn signal on. What is the probability that he is turning left?

A. 0.2 B. 0.4 C. 0.6 D. 0.8 E. 0.3 F. 0.5 G. 0.7 H. 0.9

TABLE 1 Areas Under the Standard Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3820
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998