MATH 115 - FINAL EXAM

December 14, 2011

Name _____

Student no. (from ID)_____

Rec. Day & time _____

This is a Multiple choice, closed book, no calculator exam. You may use a $5'' \times 8''$ card. Show all your work.

PUT YOUR ANSWERS ON THE ANSWER SHEET (page 19). Next to the number for each problem circle your letter answer (for example if your answer to problem 11 is A circle the letter A next to 11 on the answer sheet). Make sure that you give an answer for each question and that you have circled the letter that you intended to!

There is a table for the area under the standard normal distribution at the end of the exam that you may need.

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this (examination)(paper).

Name (printed) _____

Signature _____

Date _____

1. X is a continuous random variable on the interval [0,1] whose density function is of the form $k(1-x)$ for some constant k. What is Var(X)?

   A. 1/2 B. 1/3 C. 1/6 D. 1/9 E. 1/12 F. 1/18 G. $\sqrt{2}/6$ H. $\sqrt{3}/6$

2. A Geiger counter clicks, on the average, every 20 seconds. (The number of clicks is a Poisson process.) It is known that during one particular minute the counter clicked at most 3 times. What is the pobability that the number of clicks during that minute was at least 2?

A. 4/13	B. 9/13	C. 12/13	D. 0
E. $4e^{-3}$	F. $1 - 4e^{-3}$	G. $13e^{-3}$	H. $1 - 13e^{-3}$

3. Find

$$\frac{\partial^2}{\partial x \partial y}\Big(x + xy - 5x^3 + y\ln(x^2+1)\Big)$$

A. $1 + \frac{1}{x^2+1}$ B. $\big(x + \ln(x^2+1)\big)\big(1 + y - 15x^2 + \frac{2xy}{x^2+1}\big)$ C. 0 D. $1 - \frac{2}{x^2+1}$

E. $\frac{2y}{x^2+1} - \frac{4x^2y}{(x^2+1)^2} - 30x$ F. $\frac{y}{x^2+1} - \frac{2xy}{(x^2+1)^2} - 30x$ G. $\frac{x^2+2x+1}{x^2+1}$ H. $\frac{x^2+3}{x^2+1}$

4. If the measurements of a and b to the nearest $1/10$ of an inch are $a = 10$ inches and $b = 16$ inches then the maximum percentage error in calculating the area $A = \pi ab$ of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is closest to:

A. $2.6\pi\%$ B. $160\pi\%$ C. $26\pi\%$ D. $13/8\%$
E. $1.60\pi\%$ F. $\pi\%$ G. $356/256000\%$ H. 50%

5. A simple economy consists of two sectors, agriculture and manufacturing. The input–output matrix is

$$A := \begin{bmatrix} .4 & .2 \\ .1 & .3 \end{bmatrix}.$$

How many units (in the form [agriculture,manufacturing]) should be produced by each sector to meet the consumer demand of 20 units agriculture and 12 units manufacturing?

A. $[25, 17]$ B. $[20, 12]$ C. $[21, 13]$ D. $[40, 24]$
E. $[41, 23]$ F. $[80, 48]$ G. $[81, 49]$ H. $[82, 50]$

6. Suppose that a random variable X is uniformly distributed on the interval $[1, 6]$. The expected value of $1/X$ is:

   A. $ln(6)$
   B. $1/5$
   C. $ln(6)/5$
   D. $ln(6)/6$
   E. $ln(5)/6$
   F. $1/6$
   G. $1/ln(6)$
   H. 0

7. Let $w = \ln(1 + \frac{x^2}{2}) - arctan(x)$ and $x = 3e^u \cos(v) + v$. Find $\frac{\partial w}{\partial u}$ at $u = v = 0$.

   A. $\frac{147}{110}$ B. $\frac{196}{110}$ C. 0 D. $\frac{49}{110}$ E. $\frac{3}{10}$ F. 3 G. $\frac{4}{10}$ H. undefined

8. The tangent plane to the ellipsoid $x^2/4 + y^2 + z^2/9 = 3$ at the point $(-2, 1, -3)$ intersects the x–axis at the point:

A. $(4, 0, 0)$
B. $(3, 0, 0)$
C. $(-3, 0, 0)$
D. $(2, 0, 0)$
E. $(6, 0, 0)$
F. $(-6, 0, 0)$
G. $(1, 0, 0)$
H. $(-1, 0, 0)$

9. Given that

$$A^2 = \begin{pmatrix} 4 & 3 \\ 9 & 7 \end{pmatrix} \quad \text{and} \quad A^3 = \begin{pmatrix} 13 & 10 \\ 30 & 23 \end{pmatrix} \quad \text{find} \quad A^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

and compute the sum of the coefficients $S = a + b + c + d$. Then $S =$

A. 4
B. -1
C. -2
D. -4
E. 0
F. 1
G. 2
H. 9

10. The maximum and minimum of $f(x, y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$ are:

A. $max = 1, min = 1$
B. $max = 1, min = 0$
C. $max = 2, min = 0$
D. $max = 2, min = -1$
E. $max = 2, min = 1$
F. $max = 3, min = 1$
G. $max = 3, min = 0$
H. $max = 3, min = 2$

11.

$$\int_1^3 \int_{x-1}^2 e^{y^2} dy dx =$$

A. 0.
B. 1.
C. e.
D. $(e^9 - e)/2$.
E. $e^9 - 1$.
F. $(e^4 - 1)/2$.
G. $e^4 - 1$.
H. $e^9 - e$.

12. Which of the following statements regarding the system of equations

$$\begin{cases} x - y + z = 1 \\ x + 2y + 3z = 4 \\ 2x + 4y + 6z = k \end{cases}$$

is true ?

A. The system has a unique solution for any value of k.
B. The system has a unique solution only when $k = 8$.
C. The system has a unique solution only when $k = 0$.
D. The system only has infinitely many solutions when $k = 8$.
E. The system infinitely many solutions for every value of k.
F. The system has a unique solution for $k = 0$ and infinitely many when $k = 8$.
G. The system never has a solution.
H. The system has infinitely many solutions when $k > 0$, a unique solution when $k = 0$, and no solutions for $k < 0$.

13. It has been determined that at a certain intersection cars arriving from the west go straight 10% of the time, turn left 70% of the time, and turn right 20% of the time. It is also known that 80% of drivers use their turn signals regularly (you can assume always) while 20% use them rarely (you can assume never). You, who are heading into the intersection from the west, are sitting behind a driver who does not have his turn signal on. What is the probability that he is turning left?

    A. 0.2 B. 0.3 C. 0.4 D. 0.5 E. 0.6 F. 0.7 G. 0.8 H. 0.9

14. $A = \begin{bmatrix} 0 & 3 & -2 \\ 2 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix}$

The sum of the entries in the third row of A^{-1} equals:

A. -2 B. 0 C. 2 D. 4 E. 8 F. 10 G. -4 H. -8

15. Suppose a committee of 8 people is selected in a random manner from 15 people. Determine the probability that two particular people, A and B, will both be selected.

A. $\frac{1}{15}$ B. $\frac{8!}{15!}$ C. $\frac{4}{15}$ D. $\frac{13!}{6!7!}\frac{8!}{15!}$ E. $\frac{7!}{15!}$ F. $\frac{13!}{6!8!}\frac{7!}{15!}$ G. $\frac{13!}{15!}$ H. $\frac{8!}{6!15}$

16. Consider three events A, B and C. Assume we know $Pr(A) = .4$, $Pr(B) = .4$, $Pr(C) = .5$, $Pr(A \cap B \cap C) = .1$, $Pr(A|C) = .4$, $Pr(B|C) = .4$, and $Pr(A|B) = .5$. Then $Pr(A \cup B \cup C)$ is:

    A. 0 B. 0.2 C. 0.3 D. 0.4 E. 0.5 F. 0.6 G. 0.8 H. 1

17. The SAT score of students in a high school has a normal distribution with mean 1000 and standard deviation 200. In order to be admitted to Prestigious University, the SAT score has to be at least 1300. If the high school has 1500 students, and they all applied to PU, the number of students expected to be admitted lies in the range

    A. $171 - 200$
    B. $91 - 101$
    C. $80 - 91$
    D. $61 - 80$
    E. $31 - 60$
    F. $151 - 171$
    G. $102 - 150$
    H. $0 - 30$

18. An economy depends on two basic products: coal and gas. In order to produce 1 unit of coal, one needs 0.5 units of coal and 0.25 units of gas. In order to produce 1 unit of gas, one needs 0.4 units of coal and 0.2 units of gas. What should the production be in order to satisfy a final demand of 2 million units of coal and 4 million units of gas ?

    A. 9.3 million units of coal and 2 million units of gas.
    B. 10.67 million units of coal and 8.33 million units of gas.
    C. 8.67 million units of coal and 6.33 million units of gas.
    D. 5.3 million units of coal and 6.33 million units of gas.
    E. 10.67 million units of coal and 4.33 million units of gas.
    F. 5.3 million units of coal and 8.33 million units of gas.
    G. 3.3 million units of coal and 8.33 million units of gas.
    H. 11.3 million units of coal and 8.33 million units of gas.

ANSWER SHEET

Problem	Answer
1)	A B C D E F G H
2)	A B C D E F G H
3)	A B C D E F G H
4)	A B C D E F G H
5)	A B C D E F G H
6)	A B C D E F G H
7)	A B C D E F G H
8)	A B C D E F G H
9)	A B C D E F G H
10)	A B C D E F G H
11)	A B C D E F G H
12)	A B C D E F G H
13)	A B C D E F G H
14)	A B C D E F G H
15)	A B C D E F G H
16)	A B C D E F G H
17)	A B C D E F G H
18)	A B C D E F G H