Math 115, Fall 2010 Nguyen, Scedrov

Final Exam

December 17, 2010

Name:_____

Penn ID #:_____

Show all your work. A correct answer without supporting work receives little or no credit!

	Full score	Your score
Problem 1	18	
Problem 2	18	
Problem 3	18	
Problem 4	18	
Problem 5	18	
Problem 6	18	
Problem 7	18	
Problem 8	18	
Problem 9	18	
Problem 10	18	
Problem 11	18	
Problem 12	18	
Problem 13	18	
Problem 14	18	
Problem 15	18	
Problem 16	18	
Problem 17	18	
Problem 18	18	
Problem 19	18	
Total	342	

1. $\int_0^1 \int_{\sqrt{y}}^1 e^{x^3}\, dxdy =$

(A) 1 (B) $1-e$ (C) $1-e^{-1}$ (D) $e-1$

(E) $\frac{1}{3}$ (F) $\frac{1}{3}(1-e)$ (G) $\frac{1}{3}(1-e^{-1})$ (H) $\frac{1}{3}(e-1)$

2. The function $f(x, y) = x^2 - 4x + y^3 - 3y$ has two critical points. Find the points and determine their types.

(A) local max at (2,1); local max at (2,-1)
(B) local max at (2,1); local min at (2,-1)
(C) local max at (2,1); saddle at (2,-1)
(D) local min at (2,1); local max at (2,-1)
(E) local min at (2,1); local min at (2,-1)
(F) local min at (2,1); saddle at (2,-1)
(G) saddle at (2,1); local max at (2,-1)
(H) saddle at (2,1); local min at (2,-1)
(I) saddle at (2,1); saddle at (2,-1)

3. Consider the surface $x^2 - y^2 + 2z^3 = 2$. Find the equation for the plane tangent to this surface at $(x, y, z) = (-1, -1, 1)$ and determine where the plane intersects the y-axis. The plane intersects the y-axis at $y =$

(A) $-\frac{3}{2}$ (B) 3 (C) 6 (D) $\frac{3}{2}$

(E) -1 (F) -3 (G) 1 (H) -6

4. Find the maximum of the function $f(x, y, z) = x + y + z$ on the surface whose equation is $2x^2 + 3y^2 + z^2 = 24$. Maximum=

(A) $3\sqrt{11}$ (B) $2\sqrt{11}$ (C) $\sqrt{11}$ (D) $\frac{1}{\sqrt{11}}$

(E) $\frac{1}{\sqrt{66}}$ (F) $\sqrt{66}$ (G) $\frac{\sqrt{6}}{\sqrt{11}}$ (H) $\frac{\sqrt{11}}{\sqrt{6}}$

5. There are 3 red balls, 2 green balls and 1 yellow ball in a jar. Three balls are drawn out without replacement. What is the probability that there are more red balls than yellow balls drawn?

(A) 11/20	(B) 3/5	(C) 3/4	(D) 1/2
(E) 5/8	(F) 13/20	(G) 2/3	(H) 13/24

6. A fair coin is flipped six times. What is the probability that there are more heads produced in the first three flips than in the last three flips?

(A) 1/8 (B) 3/16 (C) 1/4 (D) 5/16
(E) 11/32 (F) 3/8 (G) 7/16 (H) 13/32

7. There are four coins. One has a probability of 1/3 of producing a heads, two are fair, and one has a probability of 2/3 of producing a heads. One coin is picked at random and flipped twice, producing a heads followed by a tails. What is the probability that it is one of the unfair coins?

(A) 3/8	(B) 9/17	(C) 1/2	(D) 8/17
(E) 4/9	(F) 10/17	(G) 2/3	(H) none of these

8. There are two fair coins A and B. Both coins are tossed twice. Let N_A be the number of heads produced by coin A and let N_B be the number of heads produced by coin B. Let Y be the minimum of N_A and N_B. Find the variance of Y.

(A) 23/64 (B) 3/8 (C) 11/32 (D) 21/64
(E) 5/16 (F) 25/64 (G) 9/32 (H) none of these

9. A pair of fair dice with sides numbered 1 through 6 are rolled 10 times. For each roll the sum of the dice is noted. Find the probability that exactly three times a "4" or a "10" is rolled.

(A) $\frac{2}{3} \cdot \frac{8^7}{9^7}$ (B) $\frac{1}{9} \cdot \frac{5^7}{6^7}$ (C) $\frac{2}{3} \cdot \frac{8^8}{9^7}$ (D) $\frac{1}{3} \cdot \frac{5^8}{6^8}$

(E) $\frac{1}{9} \cdot \frac{5^8}{6^7}$ (F) $\frac{1}{3} \cdot \frac{5^7}{6^8}$ (G) $\frac{1}{3} \cdot \frac{8^7}{9^7}$ (H) none of these

10. Suppose X is a continuous random variable uniformly distributed on the interval [1,5]. Compute the expected value of $1/X^3$.

(A) 1/25 (B) 7/25 (C) 4/25 (D) 2/25
(E) 3/5 (F) 3/25 (G) 1/5 (H) none of these

11. The continuous random variables X and Y are distributed over the square $0 \leq x \leq 2$ and $0 \leq y \leq 2$ with a joint probability density ky. Compute $\Pr(Y > X | X < 1)$.

(A) 7/12 (B) 21/24 (C) 9/12 (D) 11/24
(E) 11/12 (F) 17/24 (G) 13/24 (H) none of these

12. A point (x, y) is chosen at random in a rectangle 5 feet by 3 feet. What is the probability that the two coordinates x and y are within two feet of each other, *i.e.*, compute $\Pr(|x - y| < 2)$.

(A) 2/3 (B) 7/15 (C) 1/2 (D) 4/5
(E) 1/3 (F) 8/15 (G) 11/15 (H) none of these

13. The number of clicks of a Geiger counter is a Poisson process with a mean of one click per second. In a given period of two seconds it is known that there are less than four clicks. What is the expected number of clicks in that period?

(A) 38/21	(B) 30/19	(C) 37/21	(D) 32/19
(E) 30/21	(F) 38/19	(G) 32/21	(H) none of these

14. The random variables X and Y are independent exponentially distributed random variables, each with mean of one second. Compute the probability that Y occurs more than one second after X, $\Pr(Y > X + 1)$. To get credit you must set up and evaluate the integral.

(A) $2e^{-1}$ (B) $-\frac{3}{2}e^{-1}$ (C) $\frac{5}{2}e^{-1}$ (D) e^{-1}
(E) $\frac{3}{2}e^{-1}$ (F) $-2e^{-1}$ (G) $-e^{-1}$ (H) none of these

15. Consider the equations $2x - y = 1$ and $2x - Ax = B$, where A and B are the numbers obtained by rolling two fair dice, each numbered 1 through 6. What is the probability that these equations have at least one solution?

(A) 1/4	(B) 31/36	(C) 1/12	(D) 3/4
(E) 29/36	(F) 5/36	(G) 11/12	(H) none of these

16. If R is an invertible 3×3 matrix such that $R \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} c \\ a \\ -b \end{bmatrix}$, then compute $R^{-1} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

(A) $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$ (B) $\begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}$ (C) $\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ (D) $\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$

(E) $\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$ (F) $\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$ (G) $\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$ (H) none of these

17. For a special appearance of A Famous Pianist a total of $2,000$ tickets were sold. Front orchestra seats cost \$80 each, rear orchestra seats cost \$60 each, and front balcony seats cost \$50 each. The total receipts for the appearance were \$125,600. The combined number of tickets sold for the front orchestra and rear orchestra exceeded twice the number of front balcony seats by 800. Determine how many tickets were sold for the rear orchestra seats.

(A) $1,080$ (B) 510 (C) $1,120$ (D) 460
(E) 500 (F) $1,050$ (G) 520 (H) none of these

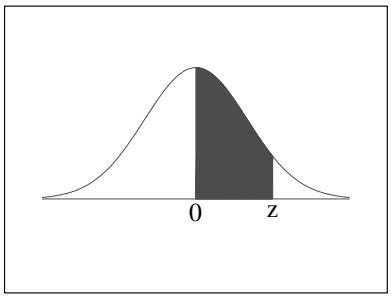
18. Ben, Jack and Zach are playing catch. Ben is as likely to throw to Jack as he is to throw to Zach. Jack is three times more likely to throw to Zach than he is to throw to Ben. Zach is as likely to throw to Ben as he is to throw to Jack. What is the probability that Jack will have the ball in the long run? To get credit you must set up the transition matrix and solve the corresponding linear system.

(A) 1/9	(B) 2/9	(C) 1/2	(D) 2/5
(E) 5/18	(F) 1/3	(G) 7/18	(H) none of these

19. Students in a large class take two midterms whose scores are approximately normally distributed. The first midterm has a mean of 75 and a standard deviation of 3. The second midterm has a mean of 85 and a standard deviation of 4. Find the approximate probability that the sum of the two scores for a student chosen at random will be larger than 165. Circle the closest answer. Indicate what you looked up and how you used it.

(A) 5%	(B) 10%	(C) 15%	(D) 20%
(E) 25%	(F) 30%	(G) 35%	(H) 40%

Standard Normal Distribution Table



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Answer key:
HFBB FEDA EFEA BHBH CFC