Final Exam

May 4, 2012

Name:______________________________

Penn ID #:____________________________

Show all your work. A correct answer without supporting work receives little or no credit!

	Full score	Your score
Problem 1	21	
Problem 2	21	
Problem 3	21	
Problem 4	21	
Problem 5	21	
Problem 6	21	
Problem 7	21	
Problem 8	21	
Problem 9	21	
Problem 10	21	
Problem 11	21	
Problem 12	21	
Problem 13	21	
Problem 14	21	
Problem 15	21	
Problem 16	21	
Problem 17	21	
Problem 18	21	
Problem 19	21	
Total	399	

1. The function $f(x, y) = 2x^3 - 3x^2 + y^2 - 2y$ has two critical points. One of them is a local minimum, whose coordinates are (a, b). Find the sum of the coordinates $a + b$.

(A) 1	(B) -1	(C) 2	(D) -2
(E) 0	(F) $\frac{1}{2}$	(G) $-\frac{1}{2}$	(H) none of these

2. Find the shortest distance from the plane $x + 2y + 2z = 9$ to the origin.

(A) 5 (B) 4 (C) $\sqrt{3}$ (D) 9

(E) $\sqrt{2}$ (F) $\sqrt{5}$ (G) 3 (H) 2

3. Consider the surface $2x^4 + y^4 + 4z^2 = 7$. Find the equation for the plane tangent to this surface at $(x, y, z) = (1, 1, 1)$ and determine where the plane intersects the y-axis. The plane intersects the y-axis at $y =$

(A) -3	(B) 3	(C) -5	(D) 5
(E) -7	(F) 7	(G) -1	(H) 1

4. $\int_0^1 \int_{\sqrt{y}}^1 cos x^3 \, dx dy =$

(A) $\frac{1}{3} sin 1$ (B) $sin 1$ (C) $\frac{1}{2} sin 1$ (D) $-\frac{1}{3} sin 1$

(E) $\frac{1}{3}$ (F) $1 - sin 1$ (G) $-\frac{1}{3}$ (H) $-sin 1$

5. There are seven pairs of socks (red, blue, gray, white, green, orange, and yellow) and three are drawn out. What is the probability that there is a pair of socks among the three drawn out?

(A) 11/52	(B) 7/39	(C) 4/39	(D) 1/39
(E) 3/13	(F) 7/13	(G) 1/3	(H) 1/2

6. There are four coins. One has a probability of 1/3 of producing a heads, two are fair, and one has a probability of 2/3 of producing a heads. One coin is picked at random and flipped twice, producing a tails followed by a heads. What is the probability that it is one of the fair coins?

(A) 3/8	(B) 11/17	(C) 1/2	(D) 8/17
(E) 4/9	(F) 10/17	(G) 2/3	(H) none of these

7. Coin A produces heads 1/4 of the time and tails 3/4 of the time. Coin B is fair. Each coin is tossed twice. What is the probability that each coin produces the same number of heads?

(A) 1/8	(B) 3/16	(C) 1/4	(D) 5/16
(E) 3/8	(F) 11/32	(G) 7/16	(H) 13/32

8. A lottery consists of choosing 11 different numbered balls, the winning numbers, at random from a group of 100. A player chooses 11 numbers on any one ticket. Find the *approximate* expected number of winning numbers chosen on a ticket.

(A) 0.25	(B) 0.5	(C) 0.75	(D) 1
(E) 1.25	(F) 1.5	(G) 1.75	(H) 2

9. A pair of fair dice with sides numbered 1 through 6 are rolled 10 times. For each roll the sum of the dice is noted. Find the probability that exactly three times a "5" or a "11" is rolled.

(A) $12 \cdot \frac{8^7}{9^7}$ (B) $24 \cdot \frac{5^8}{6^{10}}$ (C) $6 \cdot \frac{8^8}{9^7}$ (D) $8 \cdot \frac{5^8}{6^8}$

(E) $6 \cdot \frac{5^8}{6^7}$ (F) $24 \cdot \frac{5^7}{6^8}$ (G) $12 \cdot \frac{5^7}{6^{10}}$ (H) none of these

10. A point (x, y) is chosen at random in a rectangle 4 feet by 3 feet. What is the probability that the two coordinates x and y are within one foot of each other, *i.e.*, compute $\Pr(|x - y| < 1)$.

(A) 3/8	(B) 1/2	(C) 5/8	(D) 5/12
(E) 13/24	(F) 7/12	(G) 11/24	(H) none of these

11. Suppose X and Y are independent continuous random variables uniformly distributed on the intervals $0 \leq x \leq 2$ and $0 \leq y \leq 4$, respectively. Compute the variance of $\sqrt{2}X - \frac{1}{2}Y$. Hint: First find the variance of X and the variance of Y.

(A) 1/3	(B) 1/9	(C) 4/9	(D) 2/3
(E) 1/4	(F) 3/4	(G) 4/3	(H) 1

12. The continuous random variables X and Y are distributed over the rectangle $0 \leq x \leq 1$ and $0 \leq y \leq 2$ with a joint probability density $f(x, y) = 1 - x$. Compute $\Pr(Y > X | Y < 1)$.

(A) 15/16	(B) 2/3	(C) 5/9	(D) 1/2
(E) 4/9	(F) 1/3	(G) 2/9	(H) none of these

13. The number of clicks of a Geiger counter is a Poisson process with a mean of one click per minute. In a given period of one minute it is known that there are less than three clicks. What is the expected number of clicks in that period?

(A) 13/16	(B) 9/10	(C) $1 - e^{-2}$	(D) 11/12
(E) 3/4	(F) $3e^{-2}$	(G) $\frac{8}{3}e^{-1}$	(H) none of these

14. The random variables X and Y are independent exponentially distributed random variables, each with mean of two seconds. Compute the probability that X occurs more than one second after Y, $\Pr(X > Y + 1)$. In order to get credit you must set up and evaluate the integral.

(A) $1 - 2e^{-1/2}$ (B) $1 - \frac{3}{2}e^{-1/2}$ (C) $\frac{3}{2}e^{-1/2}$ (D) $\frac{1}{2}e^{-1/2}$
(E) $1 - \frac{1}{2}e^{-1/2}$ (F) $-2e^{-1/2}$ (G) $-e^{-1/2}$ (H) none of these

15. Consider the equations $x + Ay = B$ and $2x + 2y = 4$, where A and B are the numbers obtained by rolling two fair dice, each numbered 1 through 6. What is the probability that these equations have at least one solution?

(A) 1/4	(B) 11/12	(C) 1/12	(D) 3/4
(E) 31/36	(F) 5/36	(G) 29/36	(H) none of these

16. We know $A^2 = AA = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$ and $A^3 = AAA = \begin{bmatrix} 13 & 8 \\ 8 & 5 \end{bmatrix}$. Find $A = \begin{bmatrix} \text{a} & \text{b} \\ \text{c} & \text{d} \end{bmatrix}$ and compute the sum of its entries $S = a + b + c + d$.

(A) 1/2 (B) −3/2 (C) 3/2 (D) 6
(E) 4 (F) 0 (G) 5 (H) 2

17. Max, Seiji and Grace are playing catch. Max is as likely to throw to Seiji as he is to throw to Grace. Seiji is three times more likely to throw to Grace than he is to throw to Max. Grace is as likely to throw to Max as she is to throw to Seiji. What is the probability that Grace will have the ball in the long run? In order to get credit you must set up the transition matrix and solve the corresponding linear system.

(A) 1/9	(B) 2/9	(C) 1/2	(D) 2/5
(E) 5/18	(F) 7/18	(G) 1/3	(H) none of these

18. A manufacturing company is divided into two divisions, I and II. To produce a \$1 worth of product in division I requires 40 cents spent in that division and 20 cents spent in division II. To produce a \$1 worth of product in division II requires 10 cents spent in division I and 30 cents spent in division II. How should the production levels be set in order to meet a demand for \$3 million worth of product I and \$5 million worth of product II? The production level in millions=

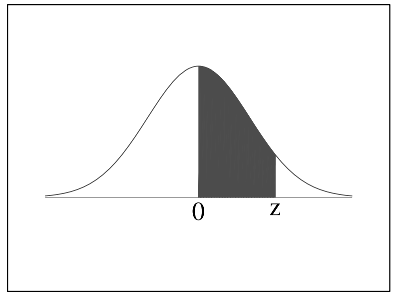
(A) $\begin{bmatrix} 7 \\ 8 \end{bmatrix}$ (B) $\begin{bmatrix} 5.5 \\ 8 \end{bmatrix}$ (C) $\begin{bmatrix} 4 \\ 6 \end{bmatrix}$ (D) $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$

(E) $\begin{bmatrix} 6.5 \\ 9 \end{bmatrix}$ (F) $\begin{bmatrix} 8 \\ 10.5 \end{bmatrix}$ (G) $\begin{bmatrix} 8 \\ 9 \end{bmatrix}$ (H) $\begin{bmatrix} 6.5 \\ 8 \end{bmatrix}$

19. A large number of exams are approximately normally distributed with mean 80 and standard deviation 10. Twenty-five exams are chosen at random. What is the probability that the average score of the chosen exams is greater than 81? Circle the closest answer. Indicate what you looked up and how you used it.

(A) 5%	(B) 10%	(C) 15%	(D) 20%
(E) 25%	(F) 30%	(G) 35%	(H) 40%

Standard Normal Distribution Table



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Answer key:
CGDA EHFE BGHB HDEG FEF