Math 115 Final Exam Monday Dec 21, 2009 Circle Instsructor Jow Powers

1. Consider the surface $z = x^4 - x + y^2 + 2y$. Find where the tangent plane plane at ($x = 1$, $y = -2$, $z = 0$) intersects the z-azis.
z-intercept = A. 2 B. $\sqrt{2}$ C. $1/\sqrt{2}$ D. 1 E. -7 F. $-\sqrt{7}$ G. $\sqrt{5}$ H. 5

2. Find the maximum of the function $f(x,y,z) = x + 2y + 3z$ on the surface $x^2 + 2y^2 + 2z^2 = 30$ Maximum =
A. 30 B. $\sqrt{30}$ C. 15 D. $\sqrt{15}$ E. 7½ F. 10 G. $2\sqrt{10}$ H. 18

3. The function $f(x,y) = 2x^3 - 3x^2 + y^2 - 2y$ has two critical points. Find the points and determine their types.

A. {rel min at x=0,y=1; rel min at x=1,y=1} B. {rel min at x=0,y=1; saddle at x=1,y=1}
C. {rel min at x=0,y=1; rel max at x=1,y=1} D. {rel max at x=0,y=1; rel min at x=1,y=1} E. {rel max at x=0,y=1; saddle at x=1,y=1}
F. {saddle at x=0,y=1; rel max at x=1,y=1} G. {saddle at x=0,y=1; rel min at x=1,y=1} H. {saddle at x=0,y=1; saddle at x=1,y=1}

4. Evaluate $\int_1^4 \int_{\frac{y-1}{3}}^1 e^{x^2}\, dx\, dy$.
A. 1 B. e C. 2 D. 1 - e E. $e^4 - 1$ F. (3/2)(e-1) G. e - 1 H. e - 2

5. Suppose the letters AABBBC in a jar are drawn out one by one. What is the probablility that the letters are drawn out in alphabetical order?
A. $\frac{1}{120}$ B. $\frac{1}{72}$ C. $\frac{1}{60}$ D. $\frac{1}{42}$ E. $\frac{2}{61}$ F. $\frac{1}{18}$ G. $\frac{1}{9}$ H. $\frac{4}{21}$

6. There are four coins A, B, C and D which land heads and tails with the probabilities shown. One of the coins is selected at random and flipped three times producing one heads and two tails. Which is the most probable coin and what is the probability it is that coin?

coin	A	B	C	D
heads	1/5	2/5	3/5	4/5
tails	4/5	3/5	2/5	1/5

Circle most probable coin. A. B. C. D.

Circle the probability it is that coin. 15% 20% 32% 36% 42% 50%

7. Six dice numbered (1,2,3,4,5,6) are tossed. Given the dice are fair and independent find the variance of the sum of the dice. Variance =
A. 17½ B. 16 C. 15½ D. 13 E. 12½ F. 10 G. 9 H. $\sqrt{37}$

8. Suppose X is distributed on the interval [-1,1] with probability distribution $f(x) = (5/2)x^4$. Find the variance of X. Var(X) =
A. 1 B. 5/4 C. 3/2 D. 5/7 E. 23/18 F. 2 G. 2½ H. 13/36

9. There are two fair coins A and B. Both coins are tossed twice. Let N_A be number of heads produced by coin A and N_B is the number of heads produced by coin B and Y is the minimum of N_A and N_B. Find the expected value of Y. $E(Y = \min(N_A, N_B)) =$

A. 1/16 B. 3/16 C. 3/8 D. 1/2 E. 5/8 F. 13/16 G. 17/16 H. 5/4

10. The random variable X is uniformly distributed over the interval [1,5]. What is the expected value of $\frac{1}{X^2}$?

A. 1/5 B. 1/3 C. e/3 D. 1/2 E. 2/π F. ln(5) G. 12/13 H. 1

11. The random variable X and Y are uniformly distributed over the rectangle $0 \leq X \leq 3$ and $0 \leq Y \leq 4$. What is the probability that $3 \leq X + Y \leq 4$?

A. 0 B. 1/12 C. 1/6 D. 1/4 E. 1/3 F. 5/12 G. 1/2 H. 2/3

12. The joint probability distribution function for X and Y where $0 \leq X \leq 1$ and $0 \leq Y \leq 2$ is $f(x,y) = 1 - x$. Compute the probability that $Y > X$ given that $Y \leq 1$. $\text{Prob}(Y > X \mid Y < 1) =$

A. 0 B. 2/9 C. 1/3 D. 4/9 E. 1/2 F. 5/9 G. 2/3 H. 15/16

13. Suppose X and Y are independent exponentially distributed random variables X with mean two seconds and Y with mean one second. Compute the probability that X occurs more than one second after Y. $\text{Prob}(X > Y+1) =$

A. $\frac{1}{2}$ B. $\frac{1}{3}$ C. $\frac{e}{2}$ D. $1-e^{-1/2}$ E. $e^{-5/3}$ F. $1-e^{-1}$ G. $\frac{2e^{-1/2}}{3}$ H. $2e^{-3/2}$

14. Two Geiger counters A and B are set to detect different radiation so they are statistically independent and the counting rate is a Poisson process with an average counting rate of two clicks per minute for A and one click per minute for B. Given that A clicks less than three times in the first minuite what is the probability A clicks less than B in the first minute. (Work out the cases where A = 0,1,2). $\text{Prob}(A<B \mid A\leq 2) =$

A. e^{-2} B. $2e^{-2}$ C. $1-e^{-2}$ D. $1-2e^{-1}$ E. $4e^{-4}$ F. $(3/4)e^{-2}$ G. $1-(7/2)e^{-3}$ H. $(7/2)e^{-3}$

15. Given the equations
$$\begin{aligned} 2x + y + 5z &= 1 \\ -x + 3y + 4z &= 0. \\ 3x - y + 3z &= 1 \end{aligned}$$
Find the value of x.

Note if $A = \begin{bmatrix} 2 & 1 & 5 \\ -1 & 3 & 4 \\ 3 & -1 & 3 \end{bmatrix}$ then $A^{-1} = \begin{bmatrix} 13 & -8 & -11 \\ 15 & -9 & -13 \\ -8 & 5 & 7 \end{bmatrix}$

A. -1 B. 0 C. 1 D. 2 E. 3 F. 4 G. 5 H. 6

16. For what values of k if any does the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 1 & 1 \\ 0 & 2 & k \end{bmatrix}$ have an inverse?

A. all values of k. B. No values of k. C. Only for k = 6. D. Only for $k \neq 6$.

E. Only for k = 1. F. Only for $k \neq 1$. G. Only for k = 2. H. Only for $k \neq 2$.

17. There are three boxes of fleas, A, B and C. Each hour a third of fleas in A hop to B, a third hop to C and a third stay in A. Each hour a third of fleas in B hop to A, a third hop to C and a third stay in B. Each hour a third of fleas in C hop to A and the remaining two thirds stay in C. If the initially A, B and C start with equal populations. What fractions of the flea population will be in B in the long run?

A. 1/5 B. 1/6 C. 2/7 D. 3/7 E. 2/5 F. 2/11 G. 7/10 H. 4/11

18. Suppose $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$ and the matrix product $AB = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$.
Find the sum
S = a + b + c + d. S = A. -2 B. -1 C. 0 D. 1 E. 2 F. 4 G. 7 H. 8

19. A wieghted coin A that produces heads 4/7 of the time and tails 3/7 of the time is tossed 49 times. A fair coin B is tossed 52 times. Using the table of the normal distribution below determine the probablility that coin A produces more heads than coin B. Show what you looked up and how you used it. Circle the closest answer.

A. 95% B. 88% C. 75% D. 66% E. 52% F. 44% G. 39% H. 21%

12345 6 7890123456789
Answers ECGFC(B 36%)ADEADGGDDHBCD

Table of the Standard Normal Distribution

$$\varphi(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-\frac{1}{2}t^2}\, dt \quad = \quad \text{Prob}(0 < Z < z)$$

a	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990